A Study of Fuzzy Assignment Method for Finding an Optimal Solution

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Introduction

In this paper, we propose a new method called fuzzy assignment method for finding a fuzzy optimal solution for a fuzzy assignment problem where all parameters are triangular fuzzy numbers. The optimal solution of fuzzy assignment problem by the fuzzy assignment method is a triangular fuzzy number. The solution procedure is illustrated with the numerical examples.

When we use the assignment method for finding an optimal solution for a fuzzy assignment problem, we have the following advantages.

- We don't use linear programming techniques.
- We don't use goal and parametric programming techniques.
- The optimal solution is a fuzzy number.
- The proposed method is very easy to understand and to apply.

Fuzzy Number and Fuzzy Assignment Problem

We use the following definitions of triangular fuzzy number and membership function and also, definitions of basic arithmetic operation on fuzzy triangular numbers.

Definition 1: A fuzzy number $\tilde{x}$ is a triangular fuzzy number denoted by (x_1, x_2, x_3) where x_1, x_2 and x_3 are real numbers and its membership function $\mu_{\tilde{x}}(a)$ is given below.

$$\mu_{\tilde{x}}(a) = \begin{cases} (a-x_1)/(x_2-x_1) & \text{for } x_1 \le a \le x_2 \\ (x_3-a)/(x_3-x_2) & \text{for } x_2 \le a \le x_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then

(1) $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$;

(2) $\tilde{a} - \tilde{b} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$;

(3) $k\tilde{a} = (ka_1, ka_2, ka_3)$, for $k \ge 0$;

(4) $k\tilde{a} = (ka_3, ka_2, ka_1)$, for $k < 0$;

(5) $\tilde{a} \otimes \tilde{b} = (t_1, t_2, t_3)$, where $t_1 = \min.\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$; $t_2 = a_2b_2$ and $t_3 = \max.\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$;

(6) $\frac{1}{\tilde{b}} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$, where b_1, b_2 and b_3 are all non zero positive real numbers.

(7) $\frac{\tilde{a}}{\tilde{b}} = \tilde{a} \otimes \frac{1}{\tilde{b}}$, where b_1, b_2 and b_3 are all non zero positive real numbers.

Definition 3: The magnitude of the triangular fuzzy number $\tilde{u} = (a, b, c)$ is given by
Mag $(\tilde{u}) = \frac{a+10b+c}{12}$.

Definition 4: Let $\tilde{u}$ and $\tilde{v}$ be two triangular fuzzy numbers.
The ranking of $\tilde{u}$ and $\tilde{v}$ by the Mag (.) on E is defined as follows:
(1) Mag$(\tilde{u})$ > Mag$(\tilde{v})$ if and only if $\tilde{u} > \tilde{v}$;
(2) Mag$(\tilde{u})$ < Mag$(\tilde{v})$ if and only if $\tilde{u} < \tilde{v}$;
(3) Mag$(\tilde{u})$ = Mag$(\tilde{v})$ if and only if $\tilde{u} = \tilde{v}$.

Fuzzy Assignment Problem
Consider the following fuzzy assignment problem,

(P) Min. $\tilde{z} = \sum_{i=1}^{n} \square \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$; [$i$ = 1, 2, 3, …, n; j = 1, 2, 3, …, n]

Subject to the constraints:

(1) $\sum_{i=1}^{n} \tilde{x}_{ij} = 1$; j = 1, 2, …, n. i.e. i^{th} person will do only one work.

(2) $\sum_{j=1}^{n} \tilde{x}_{ij} = 1$; i= 1, 2, …, n. i.e. j^{th} person will be done only one person.

Where $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3)$ = the assignment of facility i to job j
such that

$\tilde{x}_{ij}$ = 1; if i^{th} person is assigned j^{th} work
0; if i^{th} person is not assigned the j^{th} work

$\tilde{c}_{ij} = (c_{ij}^1, c_{ij}^2, c_{ij}^3)$ = the cost of assignment of resources i to activity j.
$\tilde{z} = (z_1, z_2, z_3)$ = min./max. The total cost of the matrix.

Algorithm
The fuzzy assignment method proceeds as follows:

Step 1: If the number of rows are not equal to the number of columns, the as required a dummy row or dummy column must be added. The cost elements in dummy cells are always zero.

Step 2 - Row Reduction: Subtract each row entries of the fuzzy assignment table from the corresponding row minimum.

Step 3 - Column Reduction: After completion of Step 2, subtract each column entries of the fuzzy assignment table from the corresponding column minimum.

Step 4: Remember that each row and each column now have at least one zero.

Step 5 - Zero Assignment: In the modified matrix obtained in the Step 3, search for the optimal assignment as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this row ( ) and cross off (×) all other zeros in its column. Continue in this manner until all rows have been taken care of.

(b) Repeat the procedure for each column of the reduced matrix.

(c) If a row and/or column have two or more zeros and one can not be chosen by inspection then assign arbitrary any one of the zero and cross off all other zeros of that row / column.

(d) Repeat (a) through (c) above successively until the assigning ( ) or cross (×) ends.

Step 6: If the number of assignment ( ) is equal to n (the order of the cost matrix), an optimal solution is reached.

If the number of assignment is less than n (the order of the matrix), go to the next step.

Step 7: Draw the minimum number of lines to cover zero's

In order to cover all the zero's at least once you may use the following procedure.

(1) Marks (✓) to all rows in which the assignment has not be done.

(2) See the position of zero in marked (✓) row and then mark (✓) to the corresponding column.

(3) See the marked (✓) column and find the position of assigned zero's and then mark (✓) to the corresponding rows which are not marked till now.

(4) Repeat the procedure (2) and (3) till the completion of marking.

(5) Draw the lines through unmarked rows and marked columns.

Step 8: Select the smallest element from the uncovered elements.

(a) Subtract this element from all uncovered elements and add the same to all the elements laying at the intersection of any two lines.

Step 9: Go to Step 5 and repeat the procedure until an optimum solution is attained.

Numerical Examples

The proposed method is illustrated by the following examples.

Example 1

Let three persons A, B and C are to be assigned three jobs 1, 2 and 3. The cost matrix is given as under, find the proper assignment.

Man \ Jobs	**A**	**B**	**C**
1	(1, 2, 3)	(2, 5, 8)	(2, 4, 6)
2	(2, 6, 10)	(1, 3, 5)	(0, 1, 2)
3	(4, 8, 12)	(3, 9, 15)	(1, 2, 3)

Solution

In order to find the proper assignment, we apply the fuzzy assignment method as follows:

(1) Row Reduction

Table 1

Man \ Jobs	A	B	C
1	$\tilde{0}$	(-1, -3, -5)	(-1,- 2,-3)
2	(-2, -5, -8)	(-1, -2, -3)	$\tilde{0}$
3	(-3, -6, -9)	(-2, -7, -12)	$\tilde{0}$

(2) Column Reduction

Table 2

Man \ Jobs	A	B	C
1	$\tilde{0}$	(-1, -4, -7)	(-1, -2, -3)
2	(-2, -5, -8)	(-1, -5, -9)	$\tilde{0}$
3	(-3, -6, -9)	$\tilde{0}$	$\tilde{0}$

(3) Zero Assignment

Table 3

Man \ Jobs	A	B	C
1	$\tilde{0}$	(0, 1, 2)	(1, 2,3)
2	(2, 5, 8)	(-1, -5, -9)	$\tilde{0}$
3	(3, 6, 9)	$\tilde{0}$	$\tilde{0}\times$

In this way all the zero's are either crossed out or assigned. Also total assigned zero's = 3 (i.e., number of rows or columns). Thus, the assignment is optimal.

From the table we get 1 → A; 2 → C and 3 → B.

So the minimum cost = (4, 12, 20).

Example 2

There are four machines and three jobs are to be assigned and the associated cost matrix is as follows. Find the proper assignment.

Machines \ Jobs	A	B	C	D
1	(2, 3, 4)	(3, 5, 7)	(11, 12 13)	(7, 9, 11)
2	(1, 2, 3)	(0, 1, 2)	(6, 7, 8)	(1, 2, 3)
3	(5, 7, 9)	(8, 9, 10)	(15, 17, 19)	(9, 10, 11)

Solution

In order to find the proper assignment, we apply the fuzzy assignment method as follows:

Since the no. of rows are not equal to the no. of columns so we add a dummy row with the cost zero.

Table 1

Machines \ Jobs	A	B	C	D
1	(2, 3, 4)	(3, 5, 7)	(11, 12 13)	(7, 9, 11)
2	(1, 2, 3)	(0, 1, 2)	(6, 7, 8)	(1, 2, 3)
3	(5, 7, 9)	(8, 9, 10)	(15, 17, 19)	(9, 10, 11)
4	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$

Row Reduction

Table 2

Machines \ Jobs	A	B	C	D
1	$\tilde{0}$	(-1, -2, -3)	(-9, -9, -9)	(-5, -6, -7)
2	(-1, -1, -1)	$\tilde{0}$	(-6, -6, -6)	(-1, -1, -1)
3	$\tilde{0}$	(-3, -2, -1)	(-10, -10, -10)	(-4, -3, -2)
4	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$

Zero Assignment

Table 3

Machines \ Jobs	A	B	C	D	
1	$\tilde{0}$	(-1, -2, -3)	(-9, -9, -9)	(-5, -6, -7)	✓
2	(-1, -1, -1)	$\tilde{0}$	(-6, -6, -6)	(-1, -1, -1)	
3	$\tilde{0}\times$	(-3, -2, -1)	(-10, -10, -10)	(-4, -3, -2)	✓
4	$\tilde{0}\times$	$\tilde{0}\times$	$\tilde{0}$	$\tilde{0}\times$	
	✓				

From the last table we see that all the zeros are either assigned or crossed out, but the total no. of assignment, i.e. 3 < 4. Therefore, we have to follow Step 7 and onwards as follows:

Table 4

Machines \ Jobs	A	B	C	D
1	$\tilde{0}$	(-1, -2, -3)	(-9, -9, -9)	(-5, -6, -7)
2	(-1, -1, -1)	$\tilde{0}$	(-6, -6, -6)	(-1, -1, -1)
3	$\tilde{0}\times$	(-3, -2, -1)	(-10, -10, -10)	(-4, -3, -2)
4	$\tilde{0}\times$	$\tilde{0}\times$	$\tilde{0}$	$\tilde{0}\times$

Finally, we get the assignment table as follows:

Table 5

Machines \ Jobs	A	B	C	D
1	$\tilde{0}$	(9, 8, 7)	(1, 1, 1)	(5, 4, 3)
2	(-11, -11, -11)	$\tilde{0}$	(-6, -6, -6)	(-1, -1, -1)
3	$\tilde{0}\times$	(7, 8, 9)	$\tilde{0}$	(6, 7, 8)
4	$\tilde{0}\times$	$\tilde{0}\times$	$\tilde{0}\times$	$\tilde{0}$

Here, no. of assignment = no. of columns. So this assignment tale is optimum assignment table. The assignment is as follows:

1 → A; 2 → B, 3 → C and 4 → D. So minimum cost = (17, 20, 25).

Conclusion

We have attempted to develop a new method called fuzzy zero point method to find an optimal solution to fuzzy assignment problem. The proposed method for an optimal solution is very simple, easy to understand and apply.

References

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