SOLUTION OF THE LINEAR AND NONLINEAR DIFFUSION HEAT EQUATION BY HOMOTOPY PERTURBATION METHOD

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Abstract:
He's Homotopy Perturbation Method (HAM) is powerful and efficient technique to find the solution of linear and nonlinear equation. In this paper exact solution of linear and nonlinear homogeneous diffusion equations are obtained by HPM.

Key words: HPM, Diffusion equation.

1. INTRODUCTION

The HPM method was introduced by Ji-Huan He [1-4] of Shanghai University in 1998. This method has been used by many Mathematicians and Engineers to solve various differential equations problems. HPM is a series expansion method to used in the solution of nonlinear partial differential equations. The method employs a homotopy transform to generate a convergent series solution of differential equations.

2. BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

Consider the nonlinear functional equations:

$$A(U) - f(r) = 0, \quad r \in \Omega$$

With the boundary condition:

$$B\left(U, \frac{\partial U}{\partial n}\right) = 0, \quad r \in \Gamma$$

Where A = Functional operator
B = Boundary operator
Γ =Boundary of the domain Ω

Generally A can be decomposed into two operators L and N, where L is Linear and N is nonlinear operator.

Equation (1) can be written as

$$L(U) + N(U) - f(r) = 0$$

Using the homotopy technique, we construct a homotopy $U(r, p): \Omega \times [0,1] \to R$ Which satisfies

$$H(U,P) = (1-P)\left[L(U) - L(U_0)\right] + P\left[A(U) - f(r)\right] = 0$$

OR

$$H(U,P) = L(U) - L(U_0) + PL(U_0) + P\left[N(U) - f(r)\right] = 0$$

(4)
Where

P □□0,1□is an embedding parameter, U_0 is an initial approximation of equation(1) which satisfies the boundary conditions from equations (3) and (4) we will have
H □U, 0□ □ L□U□ □ L□U_0 □ □ 0
H □U ,1□ □ A□U □ □ f □r □ □ 0

(5)
(6)
The changing process of P from Zero to unity is just that of U □r, P□from U_0 to$U$ □r □.In topology, this is called deformation, while L □U □ □ L □U_0 □and A□U □ □ f □r □ are called Homotopy.
According to HPM, we can first use the embedding parameter P as a small parameter, and assume that the solution of equations (3) and (4) can be written as a power series in P .
U □ U

□ PU

□ P^2U

□ P^3U

□ ....
(7)
0 1 2 3
Usually an approximation to the solution will be obtained by taking the limit as P □1.
U □ lim$U$ □ U_0 □U_1 □U_2 □U_3 □.................... (8)
P 1 →
Equation (8) is the solution of equation (1) obtained by HPM.

3. LINEAR HOMOGENEOUS DIFFUSION EQUATION

Consider Linear Diffusion Equation
U_t □ U_{xx} □U ,0 □ x □ 1,t □ 0
Boundary conditions are given by
U □0,t □ □ 0,U □1,t □ □ 0,t □ 0
Initial condition is given by
U □x, 0□ □ sin □ x, 0 □ x □ 1
This is a Heat equation which is solved by HPM.
U □r, P□: □□□0,1□ □ R for equation (9) is defined as (3)
(9)

(10)

(11)
□□U
□U □ □□U
□^2U □
H □U, P□ □ □1□ P □□ ___

□ □□0 □ □ P □
□ □U □ □ 0

(12)
□ □t

□t □ □ □t

□x^2 □

Suppose the solution of above Homotopy is power series in P □□0,1□ therefore equation (12) can be written as

□□□U
□ PU □ P^2U
□ ......□
□U □
□1□ P□ □ 0 1 2

□ □□0 □ □
□
□ □t
□t □

(13)

□□□U □ PU □ P^2U

□......□
□^{2} □U □ PU □ P^2U
□......□ □
P □ 0 1 2

□ 0 1 2

□ □U □ PU □P^2U □□□ □ □0
□
□ □t
□x^2
0 1 2 □
□

Comparing powers of p from both sides, we get

P^0 : □U_0 □ □U_0 □ 0

(14)
□t □t
1 □U □U □U □U □^2U
P :——— 1 □□□0 □□□0 □□□0 □□□0 □U_0 □ 0

(15)
□t □t □t □t □x^2
2 □U □U □U □^2U
P :——— 2 □□□1 □□□1 □□□1 □U_1 □ 0

(16)
□t □t □t □x^2
3 □U □U □U □^2U
P :——— 3 □□□2 □□□2 □□□2 □U_2 □ 0

(17)
□t □t □t □x^2

Solving the above partial differential equations, we get
U_0 □ sin □ x
U □ □□□ ^{2} □1□t sin$_1$□ x
(□ ^{2} □1)^2 t^2
U_2 □

sin □ x ———
2
………………………

U_3 □

□ □□ ^{2} □1□^{3} t^3 ———
6

sin □ x
solution of equation (9) can be written as
U □ U_0 □U_1 □U_2 □U_3 □ ....
2 ^{2} 2
U □ sin □ x □□□ ^{2} □1□t sin □ x □ $\dfrac{(\pi+1\ t)}{}$

sin □ x □

□□ ^{2} □1□^{3} t^3 ———

sin □ x □...............

2 6

□ □□ 2 □1□ 2 t2 □□ 2 □1□ 3 t3 □

U □ sin □ x □1□ □□ 2 □1□ t □ □ □□□

□ 2 6 □

U □ sin □ xe □□□ 2 □1□ t

This is the exact solution of (9)

4. NONLINEAR HOMOGENEOUS DIFFUSION EQUATION

Consider non homogenous Diffusion Equation

U_t □ $3U_{xx}$ □ x, 0 □ x □ □ ,t □ 0

Boundary conditions are given by

U □0,t □ □ 0,U □□ ,t □ □ 0,t □ 0

Initial condition is given by

U □x, 0□ □ sin x, 0 □ x □ □

This is a Heat equation which is solved by HPM.

U □r, P□: □□□0,1□ □ R for equation (18) is defined as (3)

(18)

(19)

(20)

□□U

□U □ □□U

$□^2U$ □

H □U,P□ □ □1□ P□□

□□□0 □ □ P □ □ 3

□ x□ □ 0

(21)

□ □t

□t □ □ □t

$□x^2$ □

Suppose the solution of above

Homotopy is power series in P □□0,1□therefore equation (21) can be written as

Comparing powers of P from both sides, we get

P^0 : □U_0 □ □U_0 □ 0

(23)

□t □t

1 □U □U □U □U □^2U

P : 1 □ □□0 □ □□0 □ □□0 □ 3 0 □ x □ 0

(24)

□t □t □t □t □x^2

2 □U □U □U □^2U

P : 2 □ □□1 □ □□1 □ 3 1 □ 0

(25)

□t □t □t □x^2

3 □U □U □U □^2U

P : 3 □ □□2 □ □□2 □ 3 2 □ 0

(26)

□t □t □t

Solving the above partial differential equations, we get

U_0 □ sin x

U_1 □ xt □ $3t$ sin x

$9t^2$

U_2 □

sin x

2

U_3 □

□$27t^3$

6

sin x

……………………………………………………

solution of equation (18) can be written as

U □ U_0 □U_1 □U_2 □U_3 □ ....

$9t^2$

$27t^3$

U □ sin x □ xt □ $3t$ sin x □

sin x □

2 6

sin x □...............

□ □$3t$□^{2} □$3t$□^{3} □

U □ xt □ sin x □1□ $3t$ □ □ □□□

2 6

$U(x,t) = \sin x\, e^{-3t}$

This is the exact solution of (18)

CONCLUSION

In this paper we have applied HPM for linear and nonlinear diffusion equations and compared with exact solutions. HPM is successful method to solve linear, nonlinear problems and gives quickly convergent.

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