Application of Double Integration: Volume of Some Special Problems of Communication System

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Abstract: **In various fields of engineering, we have to deal with the problems of evaluation of volume bounded by several complex surfaces. Double integration stands as an important mathematical tool for evaluating the volume of various complicated geometrical solids. This paper explores the process of setting up double integrals for solving some special problems of volume related with communication system with a fresh approach. The approach mentioned here demonstrate the clear overview of regions of integration, including spherical, conical and other complex shapes and offers valuable insights for both hypothetical and constructive purposes.**

Keywords: **Volume, Double Integration, Anechoic Chamber, Horn Anteena, Sphere, Cone.**

I. INTRODUCTION

Several researchers presented the study of volume in specific engineering models especially in the field of electronics engineering. J Rossignac, J J Kim, S C Song, K C Suh and C B Joung [1] have shown the boundary of the volume swept by a free-form solid in screw motion. M A Donmez, H A Inan and S S Kozat [2] have presented three robust methods to estimate an unknown signal transmitted through a time-varying flat fading channel.

A Maheshwari and B K Sharma [3] have presented an approach, according to which it is not necessary to create a two dimensional structure in one-one correspondence of the given three dimensional problem of volume estimation. Instead of it, the given problem can be solved by converting it directly in a two dimensional problem based over double integration.

In the field of communication system, anechoic chambers, horn anteena, sound propagation through human or any arbitrary source and other similar objects play very important role. It becomes very important to know about volume bounded between spherical sound propagation from any source and conic anechoic chambers to verify the amount of sound waves absorbed by anechoic chambers. Similarly, we can discuss the volume bounded between wave propagation from horn anteena with conic outer section and conic anechoic chambers.

In this paper, the approach presented by A Maheshwari and B K Sharma [3] is magnified by inclusion of one important parameter for solving some special volume problems related with several models of communication systems as mentioned above and other similar problems defined in Electronics and communication engineering or in further branches of engineering.

II. ALGORITHM

The algorithm mentioned here for solving some special volume problems is analogues to an effective technique named Nelson's technique described as "proof without words" based over geometrical analogy of algorithms, which was evolved by R. Nelsen [4].

Here, we use the observational technique to find number of different values for each of the variables x, y and z using all the surfaces given in the problem. If any of these variables has exactly two different values, then the model prescribed by A Maheshwari and B K Sharma has to be opted. But, if there is no variable with exactly two different values, then we have to pick a variable with exactly four different values in terms of two pairs of values with same expression and different signs. After that we select the appropriate formula for evaluating volume V accordingly from the list of three formulae given below:

$$V = 2\iint_{A_{xy}} \left(z_2 - z_1\right) dx\, dy \qquad \text{... (1)}$$

or $$V = 2\iint_{A_{yz}} \left(x_2 - x_1\right) dy\, dz \qquad \text{... (2)}$$

or $$V = 2\iint_{A_{xz}} \left(y_2 - y_1\right) dx\, dz \qquad \text{... (3)}$$

Above three formulae are expanded forms of the primary formula $V = \iint_D z\ dx\, dy$ mentioned by B V Ramana [5]. Two required values of variable z, x or y [as per the formula (1), (2) or (3)], must be the positive roots out of the values of pairs.

After choosing correct formula from the above mentioned list, limits for the respective double integral can be obtained using the relevant case. For example, if formula (1) is selected for evaluating volume in any problem, then limits for the respective double integral can be obtained in $xy-$plane by substituting $z = 0$ or by eliminating variable z in equations of all the surfaces given.

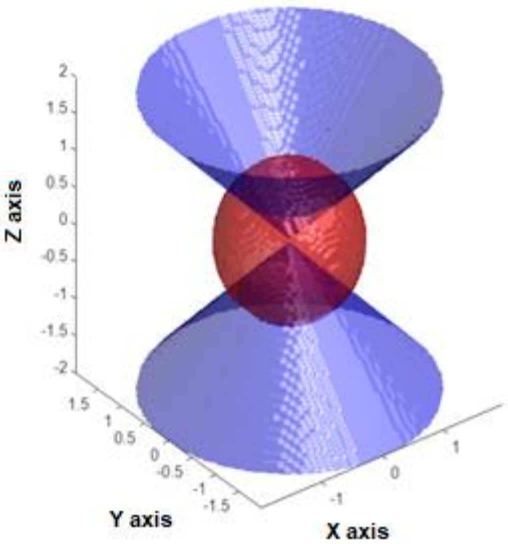
III. EXAMPLES

Example 3.1

Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ cut off by the cone $x^2 + y^2 = z^2$ using double integration.

Solution.

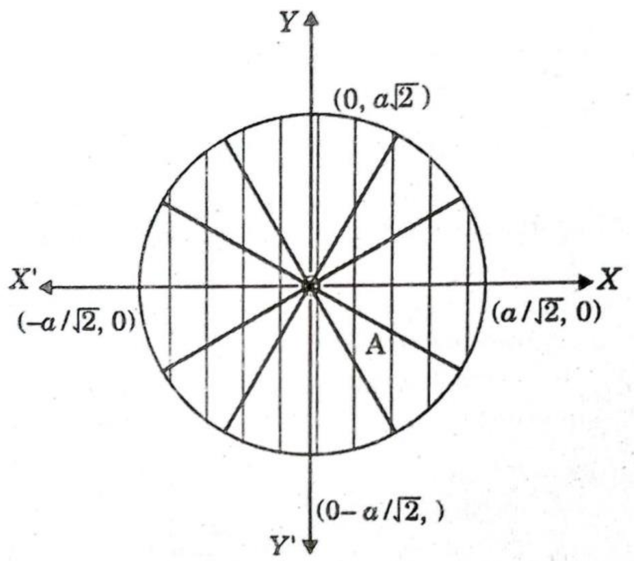
Initially if we think about figuring all surfaces given in the problem, then the following three dimensional diagram can be visualized:



Now, it seems complicated to create a mathematical model for volume evaluation by analyzing above three dimensional diagram. So, we use the algorithm mentioned in this paper. Here we observe exactly 4 values for variable x, 4 values for variable y and 4 values for variable z in pairs of positive and negative values by analyzing all the equations of the surfaces given in the problem.

Hence, any of the formula mentioned in equation (1) or equation (2) or equation (3) can be used to find the required volume. If we select equation (1) for the purpose, then two values of variable z are taken as $z_2 = \sqrt{a^2 - x^2 - y^2}$ and $z_1 = \sqrt{x^2 + y^2}$.

Hence, the required volume is given by $V = 2\iint_{A_{xy}} \left[\sqrt{a^2 - x^2 - y^2} - \sqrt{x^2 + y^2} \right] dx\,dy,$ where to find the limits in $x\,y-$plane, we eliminate variable z from all the equations of given surfaces and obtain a circle $x^2 + y^2 = \dfrac{a^2}{2}$. Region bounded by this circle (shaded by vertical as well as polar strips) can be visualized in the following diagram:



Thus, the required volume is finally obtained as:

$$V = 2\int_{x=-a/\sqrt{2}}^{a/\sqrt{2}} \int_{y=-\sqrt{\frac{a^2}{2}-x^2}}^{\sqrt{\frac{a^2}{2}-x^2}} \left[\sqrt{a^2 - x^2 - y^2} - \sqrt{x^2 + y^2} \right] dx\,dy \;= 2\int_{\theta=0}^{2\pi} \int_{r=0}^{a/\sqrt{2}} \left[\sqrt{a^2 - r^2} - r \right] r\,d\theta\,dr$$

[changing from cartesian to polar coordinates]

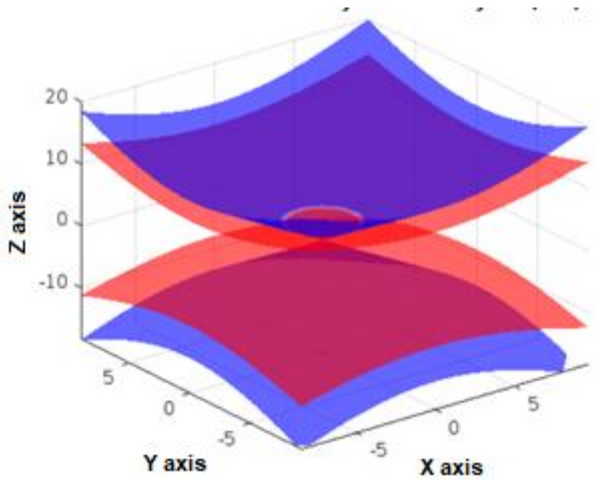
$$= 2\int_{\theta=0}^{2\pi} \left(\frac{a^3}{3} - \frac{a^3}{3\sqrt{2}} \right) d\theta = \frac{2a^3}{3}\left(1 - \frac{1}{\sqrt{2}}\right)\int_{\theta=0}^{2\pi} d\theta = \frac{4\pi a^3}{3}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{2\sqrt{2}\pi a^3}{3}\left(\sqrt{2} - 1\right)$$

Example 3.2

Find the volume bounded between the cones $4z^2 = 9x^2 + 9y^2$ and $x^2 + y^2 = (z-1)^2$ using double integration.

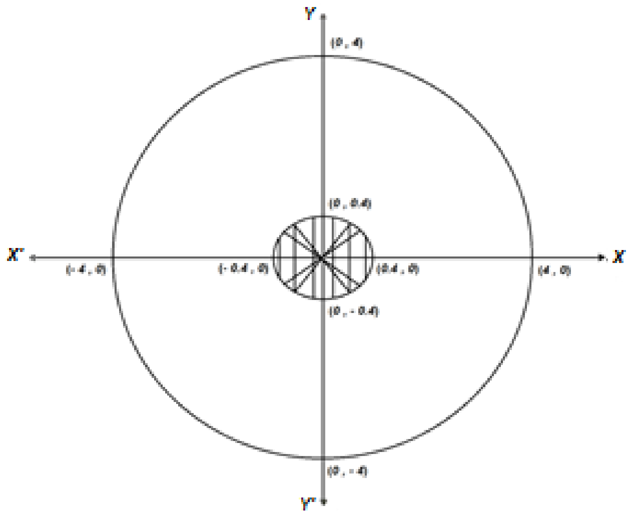
Solution.

Initially if we think about figuring both the cones given in the problem, then the following three dimensional diagram can be visualized:



Now, it seems complicated to create a mathematical model for volume evaluation by analyzing above three dimensional diagram. So, we use the algorithm mentioned in this paper. Here we observe exactly 4 values for variable x, 4 values for variable y and 4 values for variable z in pairs of positive and negative values by analyzing the equations of both the cones given in the problem. Hence, any of the formula mentioned in equation (1) or equation (2) or equation (3) can be used to find the required volume. If we select equation (1) for the purpose, then two values of variable z are taken as $z_2 = 1+\sqrt{x^2+y^2}$ and $z_1 = \frac{3}{2}\sqrt{x^2+y^2}$.

Hence, the required volume is given by $V = 2\iint_{A_{xy}} \left[\left(1+\sqrt{x^2+y^2}\right)-\left(\frac{3}{2}\sqrt{x^2+y^2}\right)\right] dx\,dy$, where to find the limits in $x\,y-$plane, we eliminate variable z from both the equations of given cones and obtain two circles $x^2+y^2 = \frac{4}{25}$ and $x^2+y^2 = 4$. Region bounded by these circles (shaded by vertical as well as polar strips) can be visualized in the following diagram:



Thus, the required volume is finally obtained as:

$$V = 2\int_{x=-2/5}^{2/5} \int_{y=-\sqrt{\frac{4}{25}-x^2}}^{\sqrt{\frac{4}{25}-x^2}} \left[\left(1+\sqrt{x^2+y^2}\right)-\left(\frac{3}{2}\sqrt{x^2+y^2}\right)\right] dx\,dy \;=2\int_{\theta=0}^{2\pi}\int_{r=0}^{2/5}\left(1-\frac{r}{2}\right) r\,d\theta\,dr$$

[changing from cartesian to polar coordinates]

$$=2\int_{\theta=0}^{2\pi}\left(\frac{r^2}{2}-\frac{r^3}{6}\right)_{r=0}^{2/5} d\theta = 2\left(\frac{2}{25}-\frac{4}{375}\right)\int_{\theta=0}^{2\pi} d\theta = \frac{104\pi}{375}$$

IV. CONCLUSION

After analyzing above examples, it becomes easy to conclude that solutions of these three-dimensional problems are conveniently obtained by the algorithm presented in this paper.

Thus, it can easily be said that the algorithm presented in this paper is very efficient and accurate, if we compare it with the method based on the complicated three-dimensional structures. Also, the designed algorithm is highly capable of solving many similar and generalized problems of Electronics and communication engineering as well as other branches of engineering.

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