COMMON INVERSE PROBLEMS FOR PHASE FIELD SYSTEM

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Abstract:
We are building the solidity results in this section for combined illustrative equations with the line between Dirichlet and phase transitions. The free boundaries problems of phase transitions have been studied for over one hundred years in the mathematical literature. Most of the work deals with the classical Stefan issue which contains a uniform medium with dormant warmth and warmth. The phase models were introduced first and various creators discussed the problem again from the start, as well as improving results thermal-dynamically; see, for instance, the work on a detailed account of the material science at the bottom of the picture and the Stefan enthalpy strategy should be expanded. Moreover, several model variants in the years under way have been explored and interesting results were obtained in the direction of presence and normality of solutions together with their reliance on the physical parameters.

Key words: Measurement, Thermal-Dynamically, Reliance.

INTRODUCTION

The linear model of phase describes phase transitions between pure material, e.g. solid and liquid, between two states is discussed in this chapter.

$$\left.\begin{aligned} &u_t + l(x)v_t - \nabla(a_1(x)\nabla u) = f_1(x), && \text{in } Q,\\ &v_t - \nabla(a_2(x)\nabla v) + b(x,t)v + c(x)u = f_2(x), && \text{in } Q,\\ &u(x,\theta) = u_\theta(x),\ v(x,\theta) = v_\theta(x), && \text{in } \Omega,\\ &u(x,t) = h_1(x,t),\ v(x,t) = h_2(x,t), && \text{on } \Sigma, \end{aligned}\right\} \quad (1.7)$$

Where is an open bounded subset of $\mathbb{R}^n$ for the dimension $\boldsymbol{n}$< 3 and with Boundary $\partial\Omega$ of class C^2. The coefficients $l, c \in L^\infty(\Omega),\ b \in L^\infty(Q)$ and for some fixed $\theta \in (0,T)$, the functions $u_\theta, v_\theta : \Omega \to \mathbb{R}$ are sufficiently regular (for instance, $h_1, h_2 : \Sigma \to \mathbb{R}$ The nonzero smooth Dirichlet boundary data $h_1, h_2 : \Sigma \to \mathbb{R}$ are kept fixed and $(f_1, f_2) \in (L^2(\Omega))^2$ Functions are specified. The unknown factor a\ and the factor are assumed to be sufficiently smooth and independent of time t.

The problem describes a model of a melting (solidification) process for 0 which is not in the Fourier phase. In the classical Stefan problem, the equation governs such a process.

$$(u + lv)_t = \nabla(a_1(x)\nabla u), \quad \text{in} \quad Q, \quad (1.8)$$

Where $u = u(x,t)$Denotes the temperature distribution of the material that can occupy the area in one or the other of the two stages (where the melting temperature is considered to be nil), thermal diffusivity and latent heat are coefficient l.

The internal energy e and the heat flow q are supposed to be given. by

$$e = u + lv$$

$$q = -a_1(x)\nabla u,$$

And that the equation of heat balance

$$e_t = -\nabla q$$

The smooth function is met V is called the phase field function; it is scaled to match v near +1 to liquid phase and v near -1 to solid phase. The interface is defined implicitly as the set of points where v is missing. We aim to have a Lipschitz stability estimate of the H1 standard, with only one observation in a limited domain, for the separation of two conceivable diffusion coefficients. An L2-weighted inequality of Carle man's type for Phase Field System solutions which are later expressed is the key to these reliability results. In the measurement of a solution over T0,T x SI and some measurement at a set time of T'€ only a single paper appears to be found if an allegorical system has an inverse problem. See where the synchronous rehabilitation between one coefficient and the initial reaction diffusion system conditions are discussed (to,T). There are, however, few literary works available for the accurate control of the illustrative systems, such as phase-field models with one control driving and a corresponding result with two control powers.

We can rapidly portray the main objective of our work as follows. Let the following system (it*, v*) be the solution

$$\left.\begin{array}{ll} u_t^* + lv_t^* - \nabla(a_1^*(x)\nabla u^*) = f_1(x), & \text{in } Q, \\ v_t^* - \nabla(a_2(x)\nabla v^*) + bv^* + cu^* = f_2(x), & \text{in } Q, \\ u^*(x,\theta) = u_\theta^*(x),\ v^*(x,\theta) = v_\theta^*(x), & \text{in } \Omega, \\ u^*(x,t) = h_1(x,t),\ v^*(x,t) = h_2(x,t), & \text{on } \Sigma. \end{array}\right\} \quad (1.9)$$

Set $p = u - u^*,\ q = v - v^*$ and $f = a_1 - a_1^*$ so that the subtraction of (1.9) from (1.8) yields

$$\left.\begin{array}{ll} p_t + lq_t - \nabla(a_1(x)\nabla p) = F, & \text{in } Q, \\ q_t - \nabla(a_2(x)\nabla q) + bq + cp = 0, & \text{in } Q, \\ p(x,\theta) = p_\theta(x),\ q(x,\theta) = q_\theta(x), & \text{in } \Omega, \\ p(x,t) = 0,\ q(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad (1.10)$$

Where $F = \nabla(f(x)\nabla u^*)$We shall make the following assumptions

Assumption 1.1 Suppose $a_1(x) \geq \mu_1 > 0,\ a_2(x) \geq \mu_2 > 0\ in\ \Omega$in Ω exists and all of their products are limited to the positive constants up to the third order respectively

$\tilde{c}_1, \tilde{c}_2\ and\ c(x) \geq c_0 > 0\ in\ \omega_2\ exists,\ where\ \omega_2 \Subset \omega \Subset \Omega.$

Dirichlet boundary data hi and h2 are regular enough and assume that the second coefficient of diffusion a2 is provided. We also assume that the measurements are restricted

q_t in Q_ω and $p(x,\theta), \nabla p(x,\theta), \Delta p(x,\theta)$ and $\nabla(\Delta p(x,\theta))$ in Ω for for some fixed $\theta \in (0,T)$, are given. The question at stake now is whether the diffusion coefficient can be determined based on the above measures.

More precisely, let (u,v) and (u^*,v^*) Be the systems solutions and be respectively the systems solutions. Then to be smooth enough*p(x,9),* there exists a constant *C* >0 depending on

$\Omega, \omega, T, \mu_1, \mu_2, \widetilde{c}_1, \widetilde{c}_2$ and c_0 satisfying

Our work gives stability results with constant diffusion coefficients for reaction diffusion systems, while our work results for phase field models with variable diffusion coefficients. Moreover, the technique we use here enables us to create two terms of stability associated with a single observation and emphasizes that there are no such results in the literature on parabolic systems. In addition, we could not get the estimate that both diffusion coefficients are simultaneously reconstructed. The problem becomes more complicated for such a reconstruction of two coefficients.

CARLE MAN ESTIMATE FOR COUPLED EQUATIONS

In this section, we present an estimate of the Carle Man type for the phase field system with a single observation on the right of the subfield u> gage. Carle men's gauge is not used specifically to show the diffusion coefficient as it shows that the gauge has different weight functions on both the left hand and the right side of the same model with a certain control limit. The creators have also used the multiplier method to identify an inequality like (1.10) with the linear zed phase field system. Nevertheless, we use a rather unique approach which covers various burdens in the main equation for dealing with the additional derived time.

Proper Weight Functions

First, two weight functions are defined that are helpful in monitoring.. First, we assume that an ip=p(x) function is found regularly, with certain characteristics defined on Q.

$$\int_\Omega (|f|^2 + |\nabla f|^2)dx \le C\int_{Q_\omega} |q_t|^2 dtdx$$
$$+C\int_\Omega \left(|\nabla p(x,\theta)|^2 + |\Delta p(x,\theta)|^2 + |\nabla(\Delta p(x,\theta))|^2\right)dx.$$

$$\psi(x) > 0 \ \ \forall x \in \Omega, \quad |\nabla\psi(x)| \ge \zeta > 0 \quad \forall x \in \bar{\Omega} \ \text{ and } \ \frac{\partial\psi}{\partial\nu} \le 0 \ \forall x \in \partial\Omega, \quad (1.11)$$

Where*v* denotes the outward normal to $\partial\Omega.$ If a function such ip can be established, the weight functions can be introduced now $\phi, \ \alpha : Q \to \mathbb{R}$

$$\phi(x,t) = e^{\lambda\psi(x)}/\beta(t) \ \text{ and } \ \alpha(x,t) = \left(e^{2\lambda\|\psi\|_{C(\bar{\Omega})}} - e^{\lambda\psi(x)}\right)/\beta(t), \quad (1.12)$$

Where $\beta(t) = t(T-t)$Note that weight an is a good weight, with t = 0 and t = T blowing up to +oo. The functions therefore $e^{-2s\alpha}, \ \phi e^{-2s\alpha}$ are smooth and they vanish at = 0 and = and also note that 0 for all $\phi(x,t) \ge C > 0$ for all e > 0 and m € R.

$$|\phi_t| \le C(\Omega)T\phi^2, \quad |\alpha_t| \le C(\Omega)T\phi^2 \ \text{ and } \ |\alpha_{tt}| \le C(\Omega)T^2\phi^3.$$

We also need the following assessments for the functions to demonstrate the main inequality (p and a: We are now denoting a generic positive constant with C'(fl), Its value varies from line to line and may vary with the ip products and S2, T. You can get the following with simple calculations

$|\phi_t| \le C(\Omega)T\phi^2, \ \ |\alpha_t| \le C(\Omega)T\phi^2$ and $|\alpha_{tt}| \le C(\Omega)T^2\phi^3.$ (1.13)Further note that

$\nabla\phi = \lambda\phi\nabla\psi, \ \nabla\alpha = -\lambda\phi\nabla\psi$ and $\phi^{-1} \le (T/2)^2$ Allow UM to be an M-bound set throughout this chapter, where M is some positive constant by

$$\mathbb{U}_M = \{k \in L^\infty(\Omega) : \|k\|_{L^\infty(\Omega)} \le M\}.$$

(1.13)Model Translation and Key Estimate

First, the problem (1.14) can be translated in the initial equation into a model without a complicated time derivative qt (note that q is the second equation solution)

$$\left.\begin{array}{ll} p_t - \nabla(a_1(x)\nabla p) + l\nabla(a_2(x)\nabla q) - c_1 p - b_1 q = F, & \text{in } Q, \\ q_t - \nabla(a_2(x)\nabla q) + bq + cp = 0, & \text{in } Q, \\ p(x,\theta) = p_\theta(x),\ q(x,\theta) = q_\theta(x), & \text{in } \Omega, \\ p(x,t) = 0,\ q(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad \textbf{(1.14)}$$

When the functions now apply to the second equation in (1.15) (referred to by (1.14) let q be the solution in (1.15) for general parable balances and assume Assumption 1.15 is true. So for everybody $\lambda \geq \bar{\lambda}_0(\Omega, T) > 0$ there exists a constant C >0 depending on Ω, ω, T, b and c and csatisfying

$$\mathcal{I}(q) \leq C\Big(\int_Q e^{-2s\alpha}|p|^2 dtdx + s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|q|^2 dtdx\Big), \quad (1.15)$$

Where ω_1 is an open set satisfying $\omega_0 \Subset \omega_1 \Subset \omega$ an

$$\mathcal{I}(q) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}(|q_t|^2 + |\Delta q|^2)dtdx + \int_Q e^{-2s\alpha}(s\lambda^2\phi|\nabla q|^2 + s^3\lambda^4\phi^3|q|^2)dtdx.$$

On the other hand, by multiplying (4.2.5) 1 by $t(T-t)$, we get

$$\left.\begin{array}{l} (t(T-t)p)_t - \nabla(a_1(x)\nabla t(T-t)p) - c_1 t(T-t)p \\ \qquad = Ft(T-t) - l\nabla(a_2(x)\nabla q)t(T-t) + b_1 t(T-t)q + p(t(T-t))_t, \\ \quad t(T-t)p = 0 \ \text{ in } \ \Sigma. \end{array}\right\} (1.16)$$

$t(T-t)p = 0$ in E.

Presently, applying the classical Carle man gauge together with the gauge (1.16) for the equation (1.16), we obtain

$$\begin{aligned} \widetilde{\mathcal{I}}(p) \ \leq\ & C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx \\ & + \int_Q e^{-2s\alpha}\phi^{-2}|l\nabla(a_2\nabla q) + b_1 q|^2 dtdx\Big), \end{aligned} \quad (1.17)$$

for $s \geq CT$ and $\lambda \geq 1$, where

$$\widetilde{\mathcal{I}}(p) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-3}(|p_t|^2 + |\Delta p|^2)dtdx + \int_Q e^{-2s\alpha}(s\lambda^2\phi^{-1}|\nabla p|^2 + s^3\lambda^4\phi|p|^2)dtdx.$$

Please note that the time factor multiplication changes the weight function forces ϕ in $\mathcal{I}(p)$New weights in Z result (p). This is now possible to estimate the last term on the right (1.17) as

$$\int_Q e^{-2s\alpha}\phi^{-2}|l\nabla(a_2\nabla q)+b_1q|^2dtdx$$
$$\leq\; 3\int_Q e^{-2s\alpha}\phi^{-2}(|l\nabla a_2\nabla q|^2+|la_2\Delta q|^2+|b_1q|^2)dtdx$$
$$\leq\; 3s^2\lambda\Big((s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}|\Delta q|^2dtdx+s\lambda^2\int_Q e^{-2s\alpha}\phi|\nabla q|^2dtdx$$
$$+s^3\lambda^4\int_Q e^{-2s\alpha}\phi^3|q|^2dtdx\Big)\leq s^2\lambda\mathcal{I}(q)$$

Provided The estimate (1.17) can now be written using the above estimate with (1.16). as

$$\widetilde{\mathcal{I}}(p)\leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2dtdx+s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2dtdx$$
$$+s^2\lambda\int_Q e^{-2s\alpha}|p|^2dtdx+s^5\lambda^5\int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|q|^2dtdx\Big)$$

Thus for any $s\geq\tilde{s}_0=\max\{\bar{s}_0,s_1,CT^2\}$ and $\lambda\geq 1$,

$$\widetilde{\mathcal{I}}(p)\leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2dtdx+s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\big(\phi|p|^2+s^2\lambda\phi^3|q|^2\big)dtdx\Big) \quad (1.18)$$

Now coupling the estimates (1.17) and (1.18), we have

$$\mathcal{I}(q)+\widetilde{\mathcal{I}}(p)\leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2dtdx$$
$$+s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\big(\phi|p|^2+s^2\lambda\phi^3|q|^2\big)dtdx\Big),$$

for the choice of $s\geq\tilde{s}_0$ and $\lambda\geq\tilde{\lambda}_0=\max\{\bar{\lambda}_0,C\sqrt{T}\}$. (1.19)

Estimation of $s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2dtdx$

In this section, we estimate the integral $|p|^2$ over Q_{ω_1} on the right-hand side of (1.19) in terms of $|q|^2$ over Q_ω to this end; first we introduce a truncating function $\chi\in C_0^\infty(\Omega)$ satisfying

$$\chi(x)=1 \text{ in } x\in\omega_1,\; 0<\chi(x)\leq 1 \text{ in } x\in\omega_2,\; \chi(x)=0 \text{ in } x\in\Omega\backslash\omega_2, \quad (1.20)$$

Where $\omega_1\Subset\omega_2\Subset\omega\Subset\Omega.$

Now we multiply the equation (1.20)

$$s^3\lambda^4\int_{Q_\omega} ce^{-2s\alpha}\phi\chi|p|^2dtdx \;=\; s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi\chi p[-q_t+\nabla(a_2\nabla q)-bq]dtdx$$
$$=\; I_1+I_2+I_3. \quad (1.21)$$

Now we estimate the components T, I = 1, 2, 3 each. Integration in integral T in parts with time, we achieve

$$\begin{aligned} I_1 &= -2s^4\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi\chi\alpha_t qp dtdx + s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi_t\chi qp dtdx \\ &\quad + s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi\chi q p_t dtdx \\ &\leq \delta_1 s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \frac{CT^2}{\delta_1}s^5\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \\ &\quad + \frac{CT^2}{\delta_1}s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi^3|q|^2 dtdx \\ &\quad + \delta_1(s\lambda)^{-1}\int_{Q_\omega} e^{-2s\alpha}\phi^{-3}|p_t|^2 dtdx + \frac{C}{\delta_1}s^7\lambda^9\int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \\ &\leq \delta_1\widetilde{I}(p) + \frac{C}{\delta_1}s^7\lambda^9\int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx, \end{aligned}$$

Whenever $\lambda \geq 1$ and $s \geq CT(1+\sqrt{T})$ and for $\delta_1 > 0$. In estimating I_2, first we observe that

$$\begin{aligned} |\nabla(\phi e^{-2s\alpha}\chi)| &= |e^{-2s\alpha}(\lambda\phi\nabla\psi\chi + 2s\lambda\phi^2\nabla\psi\chi + \phi\nabla\chi)| \\ &\leq C(\Omega,\omega)s\lambda e^{-2s\alpha}\phi^2, \quad \text{for any } s \geq CT^2 \text{ and } \lambda \geq 1. \end{aligned}$$

Similarly one can obtain

$$\begin{aligned} |\nabla(\phi e^{-2s\alpha}\chi)| &= |e^{-2s\alpha}(\lambda\phi\nabla\psi\chi + 2s\lambda\phi^2\nabla\psi\chi + \phi\nabla\chi)| \\ &\leq C(\Omega,\omega)s\lambda e^{-2s\alpha}\phi^2, \quad \text{for any } s \geq CT^2 \text{ and } \lambda \geq 1. \end{aligned}$$

Using the theorem Green and the above estimates, we have

$$\begin{aligned} I_2 &\leq \delta_2 s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \delta_2 s\lambda^2\int_{Q_\omega} e^{-2s\alpha}\phi^{-1}|\nabla p|^2 dtdx \\ &\quad + \delta_2(s\lambda)^{-1}\int_{Q_\omega} e^{-2s\alpha}\phi^{-3}|\Delta p|^2 dtdx + \frac{C}{\delta_2}s^7\lambda^8\int_{Q_\omega} e^{-2s\alpha}\phi^5|a_2|^2|q|^2 dtdx \\ &\quad + \frac{C}{\delta_2}s^7\lambda^9\int_{Q_\omega} e^{-2s\alpha}\phi^5|a_2|^2|q|^2 dtdx + \frac{C}{\delta_2}s^5\lambda^6\int_{Q_\omega} e^{-2s\alpha}\phi^3|\nabla a_2|^2|q|^2 dtdx \\ &\leq \delta_2\widetilde{I}(p) + \frac{C}{\delta_2}s^7\lambda^9\int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \end{aligned}$$

Provided $\lambda \geq C\|a_2\|^2_{L^\infty(\Omega)}$ and $s \geq CT^2\|\nabla a_2\|_{L^\infty(\Omega)}$ and for $\delta_2 > 0$.

estimating the integral I_3, we also get

$$\begin{aligned} I_3 &\leq \delta_3 s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \frac{C}{\delta_3}s^3\lambda^4\int_{Q_\omega} e^{-2s\alpha}\phi|b|^2|q|^2 dtdx \\ &\leq \delta_3\widetilde{I}(p) + \frac{1}{\delta_3}s^7\lambda^9\int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx, \end{aligned}$$

for sufficiently large $\lambda \geq 1$ and $s \geq CT^2\|b\|^{1/2}_{L^\infty(Q)}$ and for $\delta_3 > 0$. With the presumption that the c(x) and inequality coefficients are met and 6, = c0/6C for 1 < I < 3 are now read as

$$s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx$$
$$\leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx\Big), \quad (1.22)$$

For the choice of $\lambda \geq \lambda_0 = \max\{\tilde{\lambda}_0, C\|a_2\|^2_{L^\infty(\Omega)}\}$ and

$s \geq s_0 = \max\{\tilde{s}_0, CT^2(1 + 1/T + \sqrt{T}/T + \|\nabla a_2\|_{L^\infty(\Omega)} + \|b\|^{1/2}_{L^\infty(Q)})\}$.

Eventually, substituting the inequality (1.22) into (1.23), we obtain the following Carleman estimate:

Theorem 1.2.1(Carle man Estimate) Let ψ, ϕ *and* α be defined as in (1.22)-(1.23) and the coefficients l,c € UM and b € L°°(Q). Suppose Assumption 4-LI on the coefficients $a_1(x), a_2(x)$ *and* $c(x)$ Just hold true. Hold true. Then exists A0 > 0, so that the following inequality applies to all A > AQ and all s > SQ if a constant C > 0 is p and all p q satisfying p,q € is independent

$$\widetilde{\mathcal{I}}(p) + \mathcal{I}(q) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx\Big), \quad (1.23)$$

STABILITY OF THE PHASE FIELD SYSTEM

We now have an estimation of the diffuse factor as stability. The inequality of two materials (with the same geometry) estimating the difference between the a \ and a \ and Vtq and VaJ coefficients with the high limit of certain Sobolevqt solution standards and certain p-spatial derivatives at the point = 9, where 9 is a point between 0 and 0. A minimum value point of 1 fp (t). For your convenience, we refer to Q(x,9) as follows: = Q. = Q. In evidence of this stability assessment, the Carle man assessment in the previous section will be the most significant component.

Now set pt = y and qt = z, and you can write the system (1.24) as the solution with the following method (y, z):

$$\left.\begin{array}{ll} y_t + lz_t - \nabla(a_1(x)\nabla y) = F_t, & \text{in } Q, \\ z_t - \nabla(a_2(x)\nabla z) + bz + cy = 0, & \text{in } Q, \\ y(x,\theta) = \widetilde{F},\ z(x,\theta) = \widetilde{G}, & \text{in } \Omega, \\ y(x,t) = 0,\ z(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad \textbf{(1.24)}$$

where $F_t = \nabla(f\nabla u_t^*)$, $\widetilde{F} = F_\theta - lz_\theta + \nabla(a_1\nabla p_\theta)$ and $\widetilde{G} = \nabla(a_2(x)\nabla q_\theta) - bq_\theta -$

Where cp_θ. theProof of the stability gauge follows certain ideas used in the limited domain for the Faltering system and in the unlimited domain for the Schrodinger equation. We will divide the evidence into several straightforward steps in order to clarify the proof of the main outcome by proving them as preliminary results with the following norm.

Assumption 1.3.1 *If ug G C3 (fl) is a true, valuable function, ug and all its derivatives are bound and fulfilled in order of three.* $|\nabla\psi \cdot \nabla u_\theta| \geq C > 0$

Assumption 1.3.2 *Suppose* $|\Delta u^*|, |\nabla(\Delta u^*)|, |\nabla u_t^*|$ *and* $|\Delta u_t^*|$ *are bounded by a positive constant.*

Lemma 1.3.1 Consider the partial differential operator in first order, where ug meets Assumption 1-3.1. Then a constant C > 0, so that the following inequality holds sufficiently large X and s:

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|g|^2 dx \leq C \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|P_0 g|^2 dx,$$

Proof. Following the techniques of let us consider $P_0 g = \nabla u_\theta \cdot \nabla g$, $e^{-s\alpha_\theta}P_0(e^{s\alpha_\theta}w) = swP_0\alpha_\theta + P_0 w$, where we note that $g \in H_0^1(\Omega)$.

Then, by the formal integration by parts with respective to space variable, we obtain

$$\begin{aligned}
\int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx &= s^2\int_\Omega \phi_\theta^{-3}|P_0\alpha_\theta|^2|w|^2 dx + \int_\Omega \phi_\theta^{-3}|P_0 w|^2 dx \\
&\quad +2s\int_\Omega \phi_\theta^{-3} w P_0\alpha_\theta P_0 w dx \\
&= s^2\lambda^2\int_\Omega \phi_\theta^{-1}(\nabla u_\theta\cdot\nabla\psi)^2|w|^2 dx + \int_\Omega \phi_\theta^{-3}|P_0 w|^2 dx \\
&\quad -2s\lambda\int_\Omega \phi_\theta^{-2}(\nabla u_\theta\cdot\nabla\psi)(\nabla u_\theta\cdot\nabla w) w dx \\
&\geq s^2\lambda^2\int_\Omega \phi_\theta^{-1}|\nabla u_\theta\cdot\nabla\psi|^2|w|^2 dx - 2s\lambda^2\int_\Omega \phi_\theta^{-2}|P_0\psi|^2|w|^2 dx \\
&\quad +s\lambda\int_\Omega \phi_\theta^{-2}\nabla(P_0\psi\nabla u_\theta)|w|^2 dx.
\end{aligned}$$

Using Assumption 4.3.1, we obtain

$$\int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx \geq (c_1 s^2\lambda^2 - c_2 T^2 s\lambda - c_3 T^2 s\lambda^2)\int_\Omega \phi_\theta^{-1}|w|^2 dx.$$

Now for any $\lambda \geq 1$ and $s \geq 4(c_2 + c_3)T^2/c_1$, it is clear that

$$s^2\lambda^2\int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|g|^2 dx \leq C\int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx = C\int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|P_0 g|^2 dx.$$

The proof of the Lemma 1.3.1 is thus concluded.

Currently, with the assistance of Lemma 4.3.1, we demonstrate the following proposition that provides the key evaluation of the main result.

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