Some properties of Fuzzy Lattice KS Operator Group

Mr. M. U. Makandar

Assistant Professor, PG, MBA Department, KIT's IMER, Kolhapur, Maharashtra.

Abstract:
In this paper a fuzzy set is defined on a group with two operators which is also a lattice satisfying four conditions. The operator sets are denoted by K and S which are any nonempty sets. The first two conditions are according to group structure and the last two conditions are according to lattice structure.

Keywords: Lattice group, Fuzzy lattice group, Fuzzy lattice KS operator group.

INTRODUCTION

A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagrajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of fuzzy lattice KS operator group and investigated some of its basic properties.

1. PRELIMINARIES

Definition 1.1 Fuzzy group

Let λ: X to [0, 1] is a fuzzy set & (G,.) is a group which is a subset of X. A fuzzy group is a fuzzy set which satisfy two conditions

1) $\lambda(x y) \geq \text{mini}\{\lambda(x), \lambda(y)\}$
2) $\lambda(x^{-1}) \geq \lambda(x)$ where x, y∊ G .

Definition 1.2 K-Operator group

A group G is said to be an K- operator group if kx ∊ G where k∊ K (any non empty set called as Operator set) and for all x∊ G.

Definition 1.3 Fuzzy K- operator group

Let λ: X to [0, 1] is a fuzzy set & G is a subset of X which is also a K- operator group. λ is a fuzzy K-operator group if it satisfy following two conditions

i) $\lambda(k(xy)) \geq \text{mini}\{\lambda(kx), \lambda(ky)\}$
ii) $\lambda(kx)^{-1} \geq \lambda(kx)$ where x, y∊ G, k ∊ K.

Definition 1.4 Lattice K-operator group

Lattice K-operator group is an algebraic structure (G, . , R) if it satisfy two conditions 1) G is a K-operator group w.r.t '.' 2) G is a lattice w.r.t R

Definition 1.5 KS- operator group-

Let G be a group, K,S be any twononempty sets if kx ϵ G , sxϵ G. for every x ϵ G, k ϵ K, sϵ S Then G is called a KS- operator group.

Definition 1.6 Fuzzy KS- operator group

If λ: X to [0, 1] is a fuzzy set & G is KS- operator group . A fuzzy set λ over G, G subset of X is a fuzzy KS operator group if

1) $\lambda (kxsy)) \geq$ mini $\{\lambda (kx), \lambda (sy)\}$ 2) $\lambda (kx)^{-1} \geq \lambda (kx)$ & $\lambda (sx)^{-1} \geq \lambda (sx)$ for every x, yϵ G, k ϵ K ,s ϵ S

Definition 1.7 Lattice KS operator group

A lattice KS- operator group is an algebraic structure (G, R , .) if it satisfy two conditions 1) G is a KS- operator group w.r.t '.' 2) G is a lattice w.r.t R .

Definition 1.8 Fuzzy lattice KS- operator group (FL KS- operator group) –

λ: X to [0, 1] be a fuzzy set, Let G be a subset of X which is a lattice KS- operator group , K,S(operator sets). λ is a function over G. It is a fuzzy lattice KS- operator group if it satisfy following four conditions

1) $\lambda(kxsy) \geq$ mini $\{ \lambda (kx), \lambda (sy)\}$
2) $\lambda(kx)^{-1} \geq \lambda(kx)$ & $\lambda(sx)^{-1} \geq \lambda(sx)$
3) $\lambda(kx \vee sy) \geq$ mini$\{\lambda(kx),\lambda(sy)\}$
4) $\lambda(kx \wedge sy) \geq$ mini$\{\lambda(kx),\lambda(sy)\}$ For every x ϵ G, k ϵ K, s ϵ S

Definition 1.9 Fuzzy lattice KK -operator group

λ: X to [0, 1] is a fuzzy set; G is a K- lattice operator group, A function λ on G is said to be a fuzzy lattice KK-operator group if it satisfy following four conditions

1) $\lambda(k_1x\ k_2y) \geq$ mini$\{\lambda(k_1\ x),\lambda(\ k_2\ y)\}$
2) $\lambda(k_1\ x)^{-1} \geq \lambda(\ k_1x)$, $\lambda(k_2\ x)^{-1} \geq \lambda(\ k_2x)$,
3) $\lambda(k_1x \vee k_2y) \geq$ mini$\{\lambda(k_1\ x),\lambda(\ k_2\ y)\}$
4) $\lambda(k_1x \wedge k_2y) \geq$ mini$\{\lambda(k_1\ x),\lambda(\ k_2\ y)\}$, For all x, yϵ G , $k_1, k_2 \epsilon K$

Definition 1.10 Fuzzy lattice K^2-operator group

λ: X to [0, 1] is afuzzy set; G is a K- lattice operator group, A function λ on G is said to be a fuzzy lattice K-operator group if it satisfy following four conditions

1) $\lambda(kxky) \geq$ mini$\{\lambda(kx),\lambda(ky)\}$
2) $\lambda(kx)^{-1} \geq \lambda(kx)$
3) $\lambda(kx \vee ky) \geq$ mini$\{\lambda(kx),\lambda(ky)\}$
4) $\lambda(kx \wedge ky) \geq$ mini$\{\lambda(kx),\lambda(ky)\}$ For all x, y ϵ G , kϵ K

Definition 1.11 Let λ : X to Y be a function. Q is a fuzzy group of Y. A fuzzy set λ^{-1} Inverse image of Q under λ is given by $\lambda^{-1}(Q) = \mu_{\lambda^{-1}(Q)}(x) = \mu_Q \lambda (x)$

Definition 1.12 μ_A: X to [0, 1] be a fuzzy set and λ: X to X' is a function. A functionμ_{A^λ} : X to [0,1] is defined by $\mu_{A^\lambda} (x) = \mu_A\ \lambda (x)$

Definition 1.13 If T and T' are lattice KS- operator groups . A function λ: T→T' be a lattice KS homomorphism if $\lambda(k\,xsy) = \lambda(k\,x)\lambda(s\,y) = k\lambda(x)s\lambda(y)$, $\lambda(k\,x \vee sy) = \lambda(k\,x) \vee \lambda(s\,y) = k\lambda(x) \vee s\lambda(y)$, $\lambda(k\,x \wedge sy) = \lambda(kx) \wedge \lambda(sy) = k\lambda(x) \wedge s\lambda(y)$ For all x , y ϵ G, k ϵ K, s ϵ S

Definition 1.14 Let A_i be a fuzzy lattice KS operator group of G_i, for i = 1,2,……,n. Then the product A_i(i = 1,2,…..n) is the function $A_1 x A_2 x \ldots\ldots x A_n : G_1 x G_2 x \ldots\ldots x G_n \to [0,1]$ defined by$(A_1 x A_2 x \ldots\ldots x A_n)\ k\ (x_1, x_2, \ldots\ldots, x_n) =$ mini$\{A_1 (k x_1), A_2(kx_2), \ldots\ldots, A_n(kx_n)\}$

2 PROPERTIES OF FL KS- OPERATOR GROUP

Preposition 2.1: Let T and T' be two Lattice KS operator groups and λ:T to T' be a lattice KS homomorphism. If P is a FL KS operator group of T then the fuzzy set

$P^\lambda = \{ < x\ ; \mu_{P^\lambda} (x) = \mu_P\ \lambda (x) >, x \epsilon T \}$ is a FL KS operator group of T.

Proof- Let x, y ϵ T

i) $\mu_{P^\lambda}(kxsy)) = \mu_P\lambda(kxsy) = \mu_P k\lambda(x)s\lambda(y)$

$= \mu_P k\lambda(x)s\lambda(y) \geq$ mini$\{\mu_P k\lambda(x), \mu_P s\lambda(y)\}$

$\geq$ mini$\{\mu_P\ \lambda(kx), \mu_P\lambda(sy)\} \geq$ mini$\{\mu_{P^\lambda}(kx), \mu_{P^\lambda}(sy)\}$

ii) $\mu_{P^\lambda}((kx)^{-1})$ $=\mu_P \lambda[(kx)]^{-1}=\mu_P [\lambda(kx)]^{-1}$
$=\mu_P(k\lambda(x))^{-1}\geq\mu_P(k\lambda(x))=\mu_P(\lambda(kx))=\mu_{P^\lambda}(kx)$
$\mu_{P^\lambda}((sx)^{-1})$ $=\mu_P \lambda[(sx)]^{-1}=\mu_P [\lambda(sx)]^{-1}=\mu_P(s\lambda(x))^{-1}\geq\mu_P(s\lambda(x))$
$=\mu_P(\lambda(sx))=\mu_{P^\lambda}(sx)$
iii) $\mu_{P^\lambda}(kx\vee sy)$ $=\mu_P\lambda(kx\vee sy)=\mu_P\lambda(kx)\vee\lambda(sy)$
$=\mu_P k\lambda(x)\vee s\lambda(y)\geq$ mini $\{\mu_P k\lambda(x), \mu_P s\lambda(y))=$mini$\{\mu_P\lambda(kx),\mu_P\lambda(sy))=$ mini$\{\mu_{P^\lambda}(kx),\mu_{P^\lambda}(sy))$
iv) $\mu_{P^\lambda}(kx\wedge sy)$ $=\mu_P\lambda(kx\wedge sy)=\mu_P k\lambda(x)\wedge s\lambda(y)$
$\geq$ mini$\{\mu_P k\lambda(x),\mu_P s\lambda(y))=$ mini$\{\mu_P\lambda(kx),\mu_P\lambda(sy))$
$=$ mini$\{\mu_{P^\lambda}(kx),\mu_{P^\lambda}(sy)\}$

Therefore P^λ is a FL KS operator group of T.

Preposition 2.2: If λ: T to T' be a surjective lattice KS homomorphism and let P be a fuzzy lattice KS operator group of T. Define a fuzzy set P^λ: T'to [0,1] by
$P^\lambda(x')=$ mini$\{P(x)/x\epsilon\lambda^{-1}(x')\}$.Then P^λ is a fuzzy lattice KS operator group on T'.

Proof- λ: T→ T' be a surjective homomorphism and P be a fuzzy lattice operator KS group of T. Let x', y' ϵ T' and $x_0\epsilon\lambda^{-1}(x')$,$y_0\epsilon\lambda^{-1}(y')$
i) $P^\lambda(kx'sy')$ $=$ mini$\{P(z)/z\,\epsilon\lambda^{-1}(kx'sy')\}$
$\geq$mini$\{P(kx_0sy_0)/kx_0\,\epsilon\lambda^{-1}(kx'),sy_0\epsilon\lambda^{-1}(by')\}$
$\geq$mini$\{$mini$\{P(kx_0),P(sy_0))/kx_0\,\epsilon\lambda^{-1}(kx'),sy_0\epsilon\lambda^{-1}(sy')\}$
$\geq$ mini$\{$mini$\{P(kx_0)/kx_0\,\epsilon\lambda^{-1}(kx')\}$,mini$\{P(sy_0)/sy_0\epsilon\lambda^{-1}(sy')\}\}$
$=$ mini$\{P^\lambda(kx'),P^\lambda(sy')\}$
ii) $P^\lambda(kx')^{-1}$ $=$ mini$\{P(z)/z\epsilon\lambda^{-1}(kx')^{-1}\}$
$=$ mini$\{P(kx_0)^{-1})/(kx_{0)}{}^{-1}\,\epsilon\lambda^{-1}(kx')^{-1})\}$
$=$ mini$\{P(kx_0)^{-1})/kx_0\,\epsilon\lambda^{-1}(kx')\}$
$\geq$ mini$\{P(kx_0)/kx_0\,\epsilon\lambda^{-1}(kx')\}=P^\lambda(kx')$
$P^\lambda(sx')^{-1}$ $=$ mini$\{P(z)/z\epsilon\lambda^{-1}(sx')^{-1}\}=$ mini$\{P(sx_0)^{-1})/(sx_{0)}{}^{-1}\,\epsilon\lambda^{-1}(sx')^{-1})\}$
$=$ mini$\{P(sx_0)^{-1})/sx_0\,\epsilon\lambda^{-1}(sx')\}\geq$ mini$\{P(sx_0)/sx_0\,\epsilon\lambda^{-1}(sx')\}$
$=P^\lambda(sx')$
iii) $P^\lambda(kx'\vee sy')$ $=$ mini$\{P(z)/z\,\epsilon\lambda^{-1}(kx'\vee sy')\}$
$\geq$ mini$\{P(z)/z\epsilon\lambda^{-1}(kx')\vee\lambda^{-1}(sy')\}$
$\geq$ mini$\{P(kx_0\vee sy_0)/kx_0\,\epsilon\lambda^{-1}(kx'),sy_0\epsilon\lambda^{-1}(sy')\}$
$\geq$ mini$\{$mini$\{P(kx_0),P(sy_0))/kx_0\,\epsilon\lambda^{-1}(ak),by_0\epsilon\lambda^{-1}(sy')\}$
$\geq$ mini$\{$mini$\{P(kx_0)/kx_0\,\epsilon\,\lambda^{-1}(kx')\}$,mini$\{P(sy_0)/sy_0\epsilon\lambda^{-1}(sy')$
$\geq$ mini$\{P^\lambda(kx'),P^\lambda(sy')\}$
iv) $P^\lambda(kx'\wedge sy')$ $=$ mini$\{P(z)/z\epsilon\lambda^{-1}(kx'\wedge sy')\}$
$\geq$ mini$\{P(z)/z\epsilon\lambda^{-1}(kx')\wedge\lambda^{-1}(sy')\}$
$\geq$ mini$\{P(kx_0\wedge sy_0)/ax_0\,\epsilon\lambda^{-1}(kx'),sy_0\epsilon\lambda^{-1}(sy')\}$
$\geq$ mini$\{$mini$\{P(kx_0),P(sy_0)\}/kx_0\,\epsilon\lambda^{-1}(kx'),sy_0\epsilon\lambda^{-1}(sy')\}$
$\geq$ mini$\{$mini$\{P(kx_0)/kx_0\,\epsilon\lambda^{-1}(kx')\}$,mini$\{P(sy_0)/sy_0\epsilon\lambda^{-1}(sy')$
$\geq$ mini$\{P^\lambda(kx'),P^\lambda(sy')\}$
Therefore P^λ is a fuzzy lattice KS operator group on T'.

Preposition 2.3: Let λ: T→T' be a lattice KS homomorphism and P' be a fuzzy lattice KS operator group of T' then $\lambda^{-1}(P')$ is a fuzzy lattice KS operator group of T.

Proof - Let x, y ϵ T and P' be a fuzzy lattice KS operator group of T'.
i) $\lambda^{-1}(P')(kxsy)$ $=P'\lambda(kxsy)=P'(\lambda(kx)\lambda(sy))$
$=P'(k\lambda(x)s\lambda(y))=P'(k\lambda(x)s\lambda(y))$
$\geq$ mini$\{P'(k\lambda(x)), P'(s\lambda(y))\}\geq$ mini$\{P'(\lambda(kx)),P'(\lambda(sy)\}$
$\geq$ mini$\{\lambda^{-1}(P')(kx),\lambda^{-1}(P')(sy)\}$
ii) $\lambda^{-1}(P')(kx)^{-1}$ $=P'\lambda[(kx)]^{-1}=P'[\lambda(kx)]^{-1}$
$=P'[k\lambda(x)]^{-1}\geq P'(k\lambda(x))=P'(\lambda(kx))$
$\lambda^{-1}(P')(sx)^{-1}$ $=P'\lambda[(sx)]^{-1}=P'[\lambda(sx)]^{-1}=P'[s\lambda(x)]^{-1}\geq P'(s\lambda(x))$
$=P'(\lambda(sx))$

iii) $\lambda^{-1}(P')(kx \vee sy) = P'\lambda(kx \vee sy) = P'(\lambda(kx) \vee \lambda(sy))$
$= P'(k\lambda(x) \vee s\lambda(y)) \geq \text{mini}\{P'(k\lambda(x)), P'(s\lambda(y))\}$
$\geq \text{mini}\{P'(\lambda(kx)), P'(\lambda(sy)\}$
$\geq \text{mini}\ \{\lambda^{-1}(P')(kx), \lambda^{-1}(P')(sy)\ \}$
iv) $\lambda^{-1}(P')(kx \wedge sy) = P'\lambda(kx \wedge sy) = P\ (\lambda(kx) \wedge \lambda(sy))$
$= P'(k\lambda(x) \wedge s\lambda(y)) \geq \text{mini}\{P'(k\lambda(x), P'(s\lambda(y)\}$
$\geq \text{mini}\{P'(\lambda(kx)), P'(\lambda(sy))\}$
$\geq \text{mini}\{\lambda^{-1}(P')(kx), \lambda^{-1}(P')(sy)\ \}$
Therefore $\lambda^{-1}(P')$ is a fuzzy lattice KS operator group of T.

Preposition 2.4: Direct product of fuzzy lattice KS operator groups is also a fuzzy lattice KS operator group.

Proof- Let $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in G_1 x\ G_2 x \ldots x\ G_n$
Let $A_1 x\ A_2 x \ldots x\ A_n = A$
i) $A(k\ x\ s\ y)) = A(k\ (x_1, x_2, \ldots, x_n)\ s(y_1, y_2, \ldots, y_n)$
$= A\ (k\ x_1\ s\ y_1, kx_2\ sy_2, \ldots, kx_n sy_n))$
$= \text{mini}\ \{A_1\ (k\ x_1\ sy_1), A_2\ (k\ x_2\ sy_2), \ldots, A_n\ (k\ x_n\ sy_n)\ \}$
$\geq \text{mini}\{\text{mini}\ [A_1(kx_1), A_1(sy_1)], \text{min}\ [A_2\ (kx_2), A_2(sy_2)], \ldots$
$\text{mini}[\ A_n\ (k\ x_n), A_n\ (sy_n)]\ \}$
$\geq \text{mini}\ \{\text{mini}\ [A_1(k\ x_1), A_2\ (k\ x_2), \ldots A_n\ (k\ x_n)],$
$\text{mini}\ [A_1(s\ y_1), A_2\ (s\ y_2), \ldots A_n\ (s\ y_n)\ ]\}$
$\geq \text{mini}\ \{(A_1 x A_2 x \ldots x\ A_n)\ k\ (x_1, x_2, \ldots, x_n), (A_1 x\ A_2 x \ldots x\ A_n) s(y_1, y_2, \ldots, y_n)$
$\geq \text{mini}\{A(kx), A(sy)\}$
ii) $A\ (k\ x)^{-1} = A\ ((k\ x_1)^{-1}, (k\ x_2)^{-1}, \ldots, (k\ x_n)^{-1})$
$= \text{mini}\ \{A_1(k\ x_1)^{-1}, A_2(k\ x_2)^{-1}, \ldots, A_n\ (k\ x_n)^{-1})\}$
$\geq \text{mini}\ \{A_1\ (k\ x_1), A_2(k\ x_2), \ldots, A_n\ (k\ x_n)\}$
$= A\ k(x_1, x_2, \ldots, x_n)$
$\geq A\ (kx)$
$A\ (s\ x)^{-1} = A\ ((s\ x_1)^{-1}, (s\ x_2)^{-1}, \ldots, (s\ x_n)^{-1})$
$= \text{mini}\ \{A_1(s\ x_1)^{-1}, A_2(s\ x_2)^{-1}, \ldots, A_n\ (s\ x_n)^{-1})\}$
$\geq \text{mini}\ \{A_1\ (s\ x_1), A_2(s\ x_2), \ldots, A_n\ (s\ x_n)\}$
$= A\ s(x_1, x_2, \ldots, x_n)$
$\geq A\ (sx)$
iii) $A(k\ x \vee s\ y)) = A\ (k\ x_1 \vee s\ y_1, k\ x_2 \vee s\ y_2, \ldots, k\ x_n \vee sy_n))$
$= \text{mini}\ \{A_1\ (k\ x_1 \vee sy_1), A_2\ (k\ x_2 \vee sy_2), \ldots, A_n\ (k\ x_n \vee sy_n)\ \}$
$\geq \text{mini}\{\text{mini}[A_1(kx_1), A_1\ (ky_1)\ ], \text{mini}[A_2\ (kx_2), A_2(ky_2)], \ldots \text{mini}[\ A_n(kx_n), A_n(ny_n)]\ \}$
$\geq \text{mini}\{\text{mini}[A_1(kx_1), A_2(kx_2), \ldots A_n(kx_n)], \text{mini}[A_1(sy_1), A_2(sy_2), \ldots A_n(sy_n)\ ]\}$
$\geq \text{mini}\{(A_1 x A_2 x \ldots x\ A_n)\ k\ (x_1, x_2, \ldots, x_n), (A_1 x\ A_2 x \ldots x\ A_n)$
$s\ (y_1, y_2, \ldots, y_n)$
$\geq \text{mini}\{A(k\ x), A(s\ y)\}$
iv) $A(k\ x \wedge sy)) = A\ (k\ x_1 \wedge sy_1, k\ x_2 \wedge sy_2, \ldots, k\ x_n \wedge sy_n))$
$= \text{mini}\{A_1\ (k\ x_1 \wedge sy_1), A_2\ (k\ x_2 \wedge sy_2), \ldots, A_n\ (kx_n \wedge sy_n)\ \}$
$\geq \text{mini}\{\text{mini}\ [A_1(k\ x_1), A_1\ (s\ y_1)\ ], \text{mini}\ [A_2\ (k\ x_2), A_2(sy_2)], \ldots$
$\text{min}[\ A_n\ (k\ x_n), A_n\ (sy_n)]\ \}$
$\geq \text{mini}\ \{\text{mini}\ [A_1(k\ x_1), A_2\ (k\ x_2), \ldots A_n\ (k\ x_n)],$
$\text{min}\ [A_1(s\ y_1), A_2\ (s\ y_2), \ldots A_n\ (s\ y_n)\ ]\}$
$\geq \text{mini}\{(A_1 x A_2 x \ldots x\ A_n)\ k\ (x_1, x_2, \ldots, x_n), (A_1 x\ A_2 x \ldots x\ A_n)$
$s\ (y_1, y_2, \ldots, y_n)$
$\geq \text{mini}\{A(k\ x), A(s\ y)\}$

REFERENCES:

[1] Ajmal N and K.V.Thomas, The Lattice of Fuzzy subgroups and fuzzy normal sub groups, Inform. sci. 76 (1994), 1 – 11.

[2] Birkoff, G : Lattice theory 3rd edn. Amer.Math. Soc. Colloquium pub.25 (1984).
[3] G.S.V. Satya Saibaba. Fuzzy lattice ordered groups, South east Asian Bulletin of Mathematics 32,749- 766 (2008).
[4] J.A. Goguen : L – Fuzzy Sets, J. Math Anal.Appl. 18, 145-174 (1967).
[5] L. A.Zadeh : Fuzzy sets, Inform and Control, 8, 338-353 (1965).
[6] M.Marudai & V. Rajendran: Characterization of Fuzzy Lattices on a Group International Journal of Computer Applications with Respect to T-Norms, 8(8),0975 – 8887 (2010)
[7] Mordeson and D.S. Malik : Fuzzy Commutative Algebra, World Scientific Publishing Co. Pvt. Ltd.
[8] Nanda, S : Fuzzy Lattices, Bulletin Calcutta Math. Soc. 81 (1989) 1 – 2.
[9] Rosenfeld : Fuzzy groups, J. Math. Anal. Appl. 35, 512 – 517 (1971).
[10] S. Subramanian, R Nagrajan & Chellappa , Structure Properties of M-Fuzzy Groups Applied Mathematical Sciences,6(11),545-552(2012)
[11] Solairaju and R. Nagarajan : Lattice Valued Q-fuzzy left R – Submodules of Neat Rings with respect to T-Norms, Advances in fuzzy mathematics 4(2), 137 – 145 (2009).
[12] W.X.Gu. S.Y.Li and D.G.Chen, fuzzy groups with operators, fuzzy sets and system,66 (1994) ,363-3