TIME-INDEPENDENT COEFFICIENTS IN THE PHASE FIELD SYSTEM

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Abstract:
As mathematical models for the transitions of phase, phase field systems have been noted at the latest. In view of several authors (we talk of Penrose and Fife's work and take account of the exhaustive explaination of basic science), we review the phase field models first introduced, and subsequently reexamined and improved from a thermodynamic point of view, to expand the enthalpy technique on Stefan's problem to make it conceivable; In later years, mathematicians have worked incredibly hard to study several versions of the model, with interesting results in the presence and consistencies of solutions, as well as in relying on physical parameters.

Key words: Thermodynamic, Consistencies, Explanatory.

INTRODUCTION

The easiest linear phase field model can be composed as an explanatory equation system which describes transitions between two states in unadulterated material (u and v) as a solution, for instance strong or fluid.

$$\left.\begin{array}{ll} u_t + l(x)v_t - \Delta u + a(x)v = f_1(x), & \text{in } Q, \\ v_t - \Delta v + b(x,t)v + c(x)u = f_2(x), & \text{in } Q, \\ u(x,\theta)\big| = u_\theta(x),\ v(x,\theta) = v_\theta(x), & \text{in } \Omega, \\ u(x,t) = h_1(x,t),\ v(x,t) = h_2(x,t), & \text{on } \Sigma, \end{array}\right\} \quad (1.40)$$

For dimension limit 30 of class C2, open-bounded subset of R." The coefficient L € L°(0) for latent heat is b£ Cl(Q); a fixed value of some EUR 9(0,T) is adequately consistent and the semi-initial value ug is sufficiently normal (for example, ug,Vg) £(H2(O))). Non-zero smooth Dirichlet border data Hello: S —> R is kept. and $(f_1, f_2) \in (L^2(\Omega))^2$ are given functions. The response u shows the distribution of the temperature of a surface area 0, which can be strong or fluid in two stage and smooth (when the melting temperature is zero). The phase field function is called V and the purpose is to scal v almost+1 for one phase, for example the fluid phase and v near the -1 for the other high phase. The function of the phase field is clearly visible and differentiates among different phases.

It has also been identified with small amounts in numerous areas of factual mechanics. The request parameters are combined with other variables on a system with complex elements and are constrained to have a fixed value in the weight temperature plane on either end of the balance competition curve. The obscure a(x) and c(x) coefficients are considered smooth enough and are kept free from time t.

In this section, the aim is to achieve the stability estimate of Lipschitz by the internal measurements of one observation in a limited area of dimension n < 3 in de-determining the coefficients a(x) and c(x). An L2-

weighted inequality of Carle man type with solutions to phase system solutions will be the key ingredient to these stability results, which is quickly explained later. The phase field system controllability was investigated. We follow the strategy used to obtain a Carle man estimate by means of two observations in various transformations identified with inverse problems and conclude another Carle man estimate by using certain vitality typology.

In a warmth-conducting system, the first temperature and warmth coefficient were simultaneously reconstructed by Yamamoto and Zou Since Carleman's global estimates of the stability of a reverse problem concern the explanatory system, the reverse measurement of a variable coefficient and constant illustrative systems was subsequent combined with one or more reverse measures. Confrontational reconstruction of 1-solution measuring factor over (f0,T) x fi, and some measurements at certain times, T'€ and initial reaction-diffusion system conditions will take place, for example (t0, T). Conversely, all (or some) coefficients of the reaction diffusion convection system were examined by observations of arbitrary sub-specific observations over a time interval of only one component and two components at fixed positive time 0.
We wish to determine the coefficients a and c by side-dates for only a single component. Let (u, v) be the following coefficient system solutions (li, a,b,c) and the same form semiconductor border data {ue,v$)

$$\left.\begin{aligned} &\widetilde{u}_t + l(x)\widetilde{v}_t - \Delta\widetilde{u} + \widetilde{a}(x)\widetilde{v} = f_1(x), && \text{in } Q,\\ &\widetilde{v}_t - \Delta\widetilde{v} + b(x,t)\widetilde{v} + \widetilde{c}(x)\widetilde{u} = f_2(x), && \text{in } Q,\\ &\widetilde{u}(x,\theta) = \widetilde{u}_\theta(x),\ \widetilde{v}(x,\theta) = \widetilde{v}_\theta(x), && \text{in } \Omega,\\ &\widetilde{u}(x,t) = h_1(x,t),\ \widetilde{v}(x,t) = h_2(x,t), && \text{on } \Sigma. \end{aligned}\right\} \quad (1.41)$$

We set $p(x, t) = u(x, t) — u(x, t), q(x, t) = v(x, t) — v(x, t), f(x) = a(x) — a(x)$ and $g(x) = c(x) — c(x)$ so that the subtraction of (1.41) from (1.42) yields

$$\left.\begin{aligned} &p_t + l(x)q_t - \Delta p + a(x)q = fR, && \text{in } Q,\\ &q_t - \Delta q + b(x,t)q + c(x)p = g\mathcal{R}, && \text{in } Q,\\ &p(x,\theta) = p_\theta,\ q(x,\theta) = q_\theta, && \text{in } \Omega,\\ &p(x,t) = q(x,t) = 0, && \text{on } \Sigma, \end{aligned}\right\} \quad (1.42)$$

Where $R(x,t)$ —$v(x,t)$ and $1Z(x,t) = -u(x,t)$. Here and henceforth, for our convenience, we denote $((x,0) := Co$- Again by a simple computation with the transformations

$$y(x,t) = p(x,t) \quad \text{and} \quad z(x,t) = \frac{q(x,t)}{\mathcal{R}(x,t)}, \quad \text{for all } (x,t) \in Q,$$

The system (1.43) becomes

$$\left.\begin{aligned} &y_t + Kz_t - \Delta y + Az = fR, && \text{in } Q,\\ &z_t - \Delta z + Bz + D\nabla z + Ey = g, && \text{in } Q,\\ &y(x,\theta) = p_\theta, \quad z(x,\theta) = \frac{q_\theta}{\mathcal{R}_\theta}, && \text{in } \Omega,\\ &y(x,t) = z(x,t) = 0, && \text{on } \Sigma, \end{aligned}\right\} \tag{1.43}$$

Where the coefficients

$$K(x,t) = l\mathcal{R}, \quad A(x,t) = a\mathcal{R} + l\mathcal{R}_t, \quad B(x,t) = \frac{\mathcal{R}_t}{\mathcal{R}} - \frac{\Delta\mathcal{R}}{\mathcal{R}} + b,$$

$$D(x,t) = -\frac{2\nabla\mathcal{R}}{\mathcal{R}} \quad \text{and} \quad E(x,t) = \frac{c}{\mathcal{R}}.$$

Further we set ***14*** = ***yt*** and V = ***zt;*** then ***(14,*** V) satisfies

$$\left.\begin{aligned} &\mathcal{U}_t + K\mathcal{V}_t - \Delta\mathcal{U} + A\mathcal{V} + K_t\mathcal{V} + A_t z = fR_t, && \text{in } Q,\\ &\mathcal{V}_t - \Delta\mathcal{V} + B\mathcal{V} + D\nabla\mathcal{V} + E\mathcal{U} + F^{yz} = 0, && \text{in } Q,\\ &\mathcal{U}(x,\theta) = \mathcal{F}_\theta, \quad \mathcal{V}(x,\theta) = \mathcal{G}_\theta, && \text{in } \Omega,\\ &\mathcal{U}(x,t) = \mathcal{V}(x,t) = 0, && \text{on } \Sigma, \end{aligned}\right\}$$

Where

$$F^{yz} = B_t z + D_t \nabla z + E_t y$$

And the semi-initial data

$$\mathcal{F}_\theta = fR_\theta - K_\theta\mathcal{G}_\theta + \Delta y_\theta - A_\theta z_\theta$$

$$\text{and} \qquad \mathcal{G}_\theta = g + \Delta z_\theta - B_\theta z_\theta - D_\theta \nabla z_\theta - E_\theta y_\theta.$$

In order to proceed further, we make the following assumptions:

Assumption 1.1.1 *Suppose the time-independent coefficient c(x) > CQ > 0 in 0J2 exists, where u>2 <s= u* d 0.

If the source term and the factors in (1.44) meet certain conditions of smoothness and compatibility, we also have the following limitations:

Assumption 1.1.2 ***Suppose*** $|\widetilde{u}|, |\nabla\widetilde{u}|, |\Delta\widetilde{u}|, |\nabla(\Delta\widetilde{u})|, |\widetilde{u}_t|, |\widetilde{u}_{tt}|, |\nabla\widetilde{u}_t|, |\Delta\widetilde{u}_t|, |\widetilde{v}|$ *are bounded by a positive constant* $M_1 = M_1(\Omega, T) > 0$ *for all* $(x,t) \in Q$.

The Dirichlet boundary data h_l and h_2 are sufficiently regular. Also we assume that the bounded measurements z_t in Q_u and for some fixed $\theta \in (0,T)$ the terms p_θ, Δp_θ, q_θ, ∇q_θ and Δq_θ in Ωin are given. Now the question under interest is whether we can determine the time-independent coefficients ***a*** and ***c*** from the above given measurements for the cases $u_\theta = \widetilde{u}_\theta,\ v_\theta = \widetilde{v}_\theta$ and $u_\theta \neq \widetilde{u}_\theta,\ v_\theta \neq \widetilde{v}_\theta$.

More precisely, let (*u*,*v*) and *(u, v)* be the solutions of the systems (1.45) and (1.46)respectively. Then for sufficiently smooth (u,v) and $(\widetilde{u},\widetilde{v})$there exists a constant***C*** >0 depending on Ω, ω, T, M_1 and c0 satisfying

$$||a-\widetilde{a}||^2_{L^2(\Omega)} + ||c-\widetilde{c}||^2_{L^2(\Omega)} \leq C||q||^2_{H^1(0,T;L^2(\omega))},$$

For the case $u(x,\theta) = \widetilde{u}(x,\theta),\ v(x,\theta) = \widetilde{v}(x,\theta)$

$$||a-\widetilde{a}||^2_{L^2(\Omega)} + ||c-\widetilde{c}||^2_{L^2(\Omega)} \leq C\Big(||q||^2_{H^1(0,T;L^2(\omega))}$$

for the case $u(x,\theta) \neq \widetilde{u}(x,\theta),\ v(x,\theta) \neq \widetilde{v}(x,\theta)$.

We note that we have achieved stability results for a phase field code with variable diffusion coefficients in section 4, but this work establishes the after-effects for the recovery of two phase field models temporally independent with one observation. Moreover, the technique used here permits us to achieve a stability result of 2-term, 1g. It is also noteworthy that it is impossible to immediately use the technology that is used to discuss the stability results of the reaction-diffusion-convection system, as the phase field system involves time-dedicated terms in Carle Man's estimates.

STABILITY OF THE INVERSE PROBLEM

We first prove the Carle man type for a phase field system with a single observation on the right side of the assessment of a subdomain u of 0. In Part 4, Carle's men cannot evaluate a model with variable coefficients specifically by determining a diffusion coefficient with a single observation to prove two coefficients, because the estimate has been confirmed on the left and right faces of the model with different weight functions. The estimate of Carle men is based on Part 4.

Weight Functions and Notations

In the next sequence, we first define two weight functions that are useful. Let's be a fixed arbitrary sub-dome of 0. First we assume that a regular and positive function can be found with certain properties to fulfill

$$\left.\begin{array}{l}\psi(x)>0\ \ \forall x\in\Omega,\quad \psi(x)=0\ \ \forall x\in\partial\Omega,\\ |\nabla\psi(x)|>0\ \forall x\in\bar{\Omega}\backslash\omega_0 \text{ and } \frac{\partial\psi}{\partial\nu}\leq 0\ \forall x\in\partial\Omega,\end{array}\right\} \quad (1.44)$$

Where ν denotes the outward normal to 30 for the existence of such a function ψ one can refer to Now we introduce the weight functions $\lambda > 1,\ t\in(0,T)$: With A > 1, t ∈ (0, T):

$$\phi(x,t)=e^{\lambda\psi(x)}/\beta(t) \ \text{ and } \ \alpha(x,t)=\left(e^{2\lambda\|\psi\|_{C(\bar{\Omega})}}-e^{\lambda\psi(x)}\right)/\beta(t), \quad (1.45)$$

Where $\beta(t)=t(T-t)$. Note that the weight function a is positive and blows up to $+\infty$ at $t=0$ and $t=T$. as a consequence, the functions $e^{-2s\alpha}$, $\phi e^{-2s\alpha}$, etc. Are smooth and they vanish at t = 0 and t = T. Also note that $e^{-\epsilon\alpha}\phi^m\leq C<\infty$ for all $\epsilon>0$ and $m\in\mathbb{R}$. Moreover, we need the following assessments for functions p and an in order to prove the main inequalities. We will now point out a generic positive constant with C (S2), the value of which varies from line to line and depends on ip, its derivatives, and Q, T. The following estimates can be obtained by simply calculating Further note that $\phi(x,t)\geq C>0$ $(x,t)\in Q$ and $e^{-\epsilon\alpha}\phi^m\leq C<\infty$ for all $\epsilon>0$ and $m\in\mathbb{R}$. Let's say that, throughout this paper, let's have a L° A/-bound set where M is some positive constant defined by Our primary interest is to obtain an estimate from Carle Man for the {U, V) system solution in Qw, with one observation that would be used to achieve the result of the stability.

Main Results

Firstly, in the first equation (1,46), we translate the issue into a Vt time model without complicating time by properly substituting the second equation terms (remember, V is the second equation solution) as

$$\left.\begin{array}{ll}\mathcal{U}_t-\Delta\mathcal{U}+K\Delta\mathcal{V}+l_1\nabla\mathcal{V}+l_2\mathcal{V}+l_3\mathcal{U}+G^{yz}=fR_t, & \text{in } Q,\\ \mathcal{V}_t-\Delta\mathcal{V}+B\mathcal{V}+D\nabla\mathcal{V}+E\mathcal{U}+F^{yz}=0, & \text{in } Q,\\ \mathcal{U}(x,\theta)=\mathcal{F}_\theta,\ \ \mathcal{V}(x,\theta)=\mathcal{G}_\theta, & \text{in } \Omega,\\ \mathcal{U}(x,t)=\mathcal{V}(x,t)=0, & \text{on } \Sigma,\end{array}\right\} \quad (1.46)$$

Where *Fyz*, ***Fg***, ***Qg*** are defined in (1.46) and

$$G^{yz}=(A_t-KB_t)z-KD_t\nabla z-KE_ty,$$
$$l_1=-KD,\ \ l_2=A+K_t-KB,\ l_3=-KE.$$

We divide this section into two cases such as

$$(i) \quad u_\theta = \widetilde{u}_\theta, \qquad v_\theta = \widetilde{v}_\theta$$

$$(ii) \quad u_\theta \neq \widetilde{u}_\theta, \qquad v_\theta \neq \widetilde{v}_\theta.$$

We are currently required to independently establish the Carle man type assessment for case (I) and (ii) using these assessments and to Display steady estimation of Sobolev Solution Standard q of (1.46) on q on case I and on Sobolev Solution Standard q of (1.47) on Qw and spatial derivatives p and q at 9€ (0,T) on a case basis over fi (1.47) on case (1.47) (ii).

Internal Carleman Estimate for Case (i):

Step 1. We are currently applied to the second (1.45) (alluded to (1.45)2) equation, the classical calculations derived by Carleman for general illustrative equations (see example). May V (1.45)2 be the solution, and assume Assumption is true.

Then for any A > AO (0, T) > 0 and $s > s_0$(O, T, *M)* > 0, there exists a constant $C > 0$

depending on f_1, O_J, T satisfying

$$\mathcal{I}(\mathcal{V}) \leq C\Big(\int_Q e^{-2s\alpha}|E\mathcal{U} + F^{yz}|^2 dtdx + s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|\mathcal{V}|^2 dtdx\Big),$$

Where *ui* is an open set satisfying *u0 uj*\ <g *to* and

$$\mathcal{I}(\mathcal{V}) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}(|\mathcal{V}_t|^2 + |\Delta\mathcal{V}|^2)dtdx + \int_Q e^{-2s\alpha}(s\lambda^2\phi|\nabla\mathcal{V}|^2 + s^3\lambda^4\phi^3|\mathcal{V}|^2)dtdx.$$

In case (i), we note that

$$\mathcal{I}(\mathcal{V}) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}(|\mathcal{V}_t|^2 + |\Delta\mathcal{V}|^2)dtdx + \int_Q e^{-2s\alpha}(s\lambda^2\phi|\nabla\mathcal{V}|^2 + s^3\lambda^4\phi^3|\mathcal{V}|^2)dtdx.$$

Then the above estimate becomes

$$\mathcal{I}(\mathcal{V}) \leq C\Big(\int_Q e^{-2s\alpha}\Big|E\mathcal{U} + B_t\int_\theta^t \mathcal{V}(x,\tau)d\tau + D_t\int_\theta^t \nabla\mathcal{V}(x,\tau)d\tau + E_t\int_\theta^t \mathcal{U}(x,\tau)d\tau\Big|^2 dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|\mathcal{V}|^2 dtdx\Big).$$

In order to estimate the primary component on the right side, the following standard estimate is necessary.

Lemma 6.2.1 *For fixed* $\theta = \frac{T}{2}$ *and* $\alpha \in C^1(\bar{Q})$, *there exists a constant* $\alpha \in C^1(\bar{Q})$, *Satisfying*

$$\int_Q e^{-2s\alpha}\Big|\int_\theta^t \mathcal{W}(x,\tau)d\tau\Big|^2 dtdx \leq \frac{CT^2}{s}\int_Q e^{-2s\alpha}|\mathcal{W}|^2 dtdx, \quad for \quad s \geq 0.$$

Proof. The proof follows the simple ideas from . Consider the integral

$$\int_\Omega \int_0^T e^{-2s\alpha} \Big| \int_\theta^t \mathcal{W}(x,\tau)d\tau \Big|^2 dtdx$$
$$= \int_\Omega \int_0^\theta e^{-2s\alpha} \Big| \int_\theta^t \mathcal{W}(x,\tau)d\tau \Big|^2 dtdx + \int_\Omega \int_\theta^T e^{-2s\alpha} \Big| \int_\theta^t \mathcal{W}(x,\tau)d\tau \Big|^2 dtdx$$
$$= \mathcal{I}_1 + \mathcal{I}_2.$$

Using the Cauchy-Schwartz inequality, we estimate the integral I_2 as follows

$$\mathcal{I}_2 \leq \int_\Omega \int_\theta^T e^{-2s\alpha}(t-\theta)\Big(\int_\theta^t |\mathcal{W}|^2 d\tau\Big)dtdx,$$

Where $\theta = T/2$. Now from the definition of *a*, we have

$$\alpha_t = 2(t-\theta)\frac{e^{2\lambda\|\psi\|_{C(\bar{\Omega})}} - e^{\lambda\psi(x)}}{t^2(T-t)^2}.$$

We note that the function *at* $\alpha_t > 0$ for $t \in (\theta, T)$ and $e^{-2s\alpha}\alpha_t = -\frac{1}{2s}(e^{-2s\alpha})_t$. Then $\mathcal{I}_2$ becomes

$$\mathcal{I}_2 \leq -\frac{CT^2}{s}\int_\Omega \int_\theta^T (e^{-2s\alpha})_t \Big(\int_\theta^t |\mathcal{W}|^2 d\tau\Big)dtdx.$$

The time integration by parts and the fact that $e^{-2s\alpha(x,T)} = 0,$ lead to

$$\mathcal{I}_2 \leq \frac{CT^2}{s}\int_\Omega \int_\theta^T e^{-2s\alpha}|\mathcal{W}|^2 dtdx.$$

Similarly one can easily obtain the following estimate for $\mathcal{I}_1$,

$$\mathcal{I}_1 \leq \frac{CT^2}{s}\int_\Omega \int_0^\theta e^{-2s\alpha}|\mathcal{W}|^2 dtdx.$$

Coupling the estimates for *I* and I_2, one can conclude the proof.

Using Lemma 6.2.1 and Assumption we then obtain that

$$\mathcal{I}_2 \leq \int_\Omega \int_\theta^T e^{-2s\alpha}(t-\theta)\Big(\int_\theta^t |\mathcal{W}|^2 d\tau\Big)dtdx,$$

$$\mathcal{I}(\mathcal{V}) \leq C(1+s^{-1})\int_Q e^{-2s\alpha}|\mathcal{U}|^2 dtdx + Cs^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|\mathcal{V}|^2 dtdx, \quad (1.47)$$

For any $s > so$ — max $\{\bar{s}_0, CT(1+\sqrt{T})\}$ and A > 1 and the constant *C* depending on Ω, ω, M_1, M and b. and *b* on the other hand, multiplying (1.45) $\gamma(t) := [t(T-t)]^{\frac{3}{2}}$,

We obtain

$$\left.\begin{aligned}&(\gamma(t)\mathcal{U})_t + \gamma(t)(K\Delta\mathcal{V} - \Delta\mathcal{U} + l_1\nabla\mathcal{V} + l_2\mathcal{V}\\&\qquad +l_3\mathcal{U} + G^{yz}) = \gamma(t)fR_t - \gamma_t(t)\mathcal{U}, \quad \text{in} \quad Q,\\&\gamma(t)\mathcal{U} = 0, \quad \text{on} \quad \Sigma.\end{aligned}\right\} \quad (1.49)$$

Applying the classical Carle man estimate to the equation (1.48), we obtain

$$\begin{aligned}\widetilde{\mathcal{I}}(\mathcal{U}) \;\le\; & C\Big(\int_Q e^{-2s\alpha}\phi^{-3}|fR_t|^2dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}|\mathcal{U}|^2dtdx\\&+\int_Q e^{-2s\alpha}\phi^{-3}|K\Delta\mathcal{V} + l_1\nabla\mathcal{V} + l_2\mathcal{V}|^2dtdx\\&+\int_Q e^{-2s\alpha}\phi^{-3}|G^{yz}|^2dtdx\Big),\end{aligned}$$

(1.50)

for any $s \ge CT$ and $\lambda \ge 1$, where

$$\begin{aligned}\widetilde{\mathcal{I}}(\mathcal{U}) = (s\lambda)^{-1}&\int_Q e^{-2s\alpha}\phi^{-4}(|\mathcal{U}_t|^2 + |\Delta\mathcal{U}|^2)dtdx\\&+\int_Q e^{-2s\alpha}(s\lambda^2\phi^{-2}|\nabla\mathcal{U}|^2 + s^3\lambda^4|\mathcal{U}|^2)dtdx.\end{aligned}$$

For any $s \ge \widehat{s}_0 = CT^2(1+T+T^2)$ and $\lambda \ge 1$. Using Lemma 6.2.1, we have

For any $s \ge \widehat{s}_0 = CT^2(1+T+T^2)$ and $\lambda \ge 1$. Making use of the above estimations, the estimate (1.51) becomes

$$\begin{aligned}\widetilde{\mathcal{I}}(\mathcal{U}) \le C\Big(&\int_Q e^{-2s\alpha}\phi^{-3}|fR_t|^2dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}|\mathcal{U}|^2dtdx\\&+(1+s^2\lambda)\mathcal{I}(\mathcal{V}) + s^2\lambda^4\int_Q e^{-2s\alpha}|\mathcal{U}|^2dtdx\Big).\end{aligned}$$

Applying the estimate (1.51), the above estimate can now be written as

$$\begin{aligned}\widetilde{\mathcal{I}}(\mathcal{U}) \le C\Big(&\int_Q e^{-2s\alpha}\phi^{-3}|fR_t|^2dtdx\\&+s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\big(|\mathcal{U}|^2 + s^2\lambda\phi^3|\mathcal{V}|^2\big)dtdx\Big),\end{aligned}$$

Provided $s \ge \mathring{s}_0 = \max\{\widetilde{s}_0, \breve{s}_0, \widehat{s}_0, C\}$ and $\lambda \ge 1$. Now coupling the estimates (1.51) and (1.52), we have

$$\mathcal{I}(\mathcal{V}) + \widetilde{\mathcal{I}}(\mathcal{U}) \le C\Big(\int_Q e^{-2s\alpha}\phi^{-3}|fR_t|^2 dtdx$$
$$+ s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\big(|\mathcal{U}|^2 + s^2\lambda\phi^3|\mathcal{V}|^2\big)dtdx\Big),$$

For the choice of $s \ge \mathring{s}_0$ and $\lambda \ge \bar{\lambda}_0$, where the constant C depending on Q, c*o, T, M, M\, b.*

***Step 2** Estimation of* $s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}|\mathcal{U}|^2 dtdx$ Now we estimate the integral $|\mathcal{U}|^2$ over Q_{ω_1} on the right-hand side of in terms of |V|2 over Q_ω. To this end, first we introduce a cut-off function $\rho \in C_0^\infty(\Omega)$ satisfying

$\rho(x) = 1$ in $x \in \omega_1$, $0 < \rho(x) \le 1$ in $x \in \omega_2$, $\rho(x) = 0$ in $x \in \Omega\backslash\omega_2$,

Next we multiply the equation by *s3X4e~2sapU* and integrate over

$$s^3\lambda^4 \int_{Q_\omega} ce^{-2s\alpha}\rho|\mathcal{U}|^2 dtdx$$
$$= s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\rho\mathcal{U}\mathcal{R}[-\mathcal{V}_t + \Delta\mathcal{V} - B\mathcal{V} - D\nabla\mathcal{V} - F^{yz}]dtdx$$
$$= \sum_{i=1}^{5} \mathcal{K}_i. \tag{1.51}$$

Now we estimate the integrals $\mathcal{K}_i, i = 1, \cdots, 5$ one by one. Before estimating *IC* 1, we first observe by, Assumption 1.1.2, that

$$(e^{-2s\alpha}\mathcal{R})_t \le CTs\phi^2 e^{-2s\alpha},$$

For any *s* > *CT*3 Now integrating by parts with time in *JC*1, we obtain /C:

$$\mathcal{K}_1 = s^3\lambda^4 \int_{Q_\omega} (e^{-2s\alpha}\mathcal{R})_t\rho\mathcal{U}\mathcal{V}dtdx + s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\mathcal{R}\rho\mathcal{V}\mathcal{U}_t dtdx$$
$$\le \delta_1 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}|\mathcal{U}|^2 dtdx + \frac{CT^2}{\delta_1}s^5\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi^4|\mathcal{V}|^2 dtdx$$
$$+ \frac{C}{\delta_1}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^4|\mathcal{V}|^2 dtdx + \delta_1(s\lambda)^{-1} \int_{Q_\omega} e^{-2s\alpha}\phi^{-4}|\mathcal{U}_t|^2 dtdx$$
$$\le \delta_1\widetilde{\mathcal{I}}(\mathcal{U}) + \frac{C}{\delta_1}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^4|\mathcal{V}|^2 dtdx,$$

Whenever A > 1 and *s* > C_T and for ^ > 0. In estimating K_2, first we observe that

$$\begin{aligned}|\nabla(e^{-2s\alpha}\rho\mathcal{R})| &= |e^{-2s\alpha}(2s\lambda\phi\nabla\psi\rho\mathcal{R}+\nabla\rho\mathcal{R}+\rho\nabla\mathcal{R})| \\ &\leq C(\Omega,\omega,M_1)s\lambda e^{-2s\alpha}\phi, \quad \text{for any } s\geq CT^2 \text{ and } \lambda\geq 1.\end{aligned}$$

Similarly one can get

$$|\Delta(e^{-2s\alpha}\rho\mathcal{R})|\leq C(\Omega,\omega,M_1)s^2\lambda^2e^{-2s\alpha}\phi^2, \quad \text{for any } s\geq CT^2 \text{ and } \lambda\geq 1.$$

Now using Green's theorem and the above estimations, we have

$$\begin{aligned}\mathcal{K}_2 &\leq \delta_2 s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}|\mathcal{U}|^2dtdx+\delta_2 s\lambda^2\int_{Q_\omega}e^{-2s\alpha}\phi^{-2}|\nabla\mathcal{U}|^2dtdx \\ &\quad +\delta_2(s\lambda)^{-1}\int_{Q_\omega}e^{-2s\alpha}\phi^{-4}|\Delta\mathcal{U}|^2dtdx+\frac{C}{\delta_2}s^7\lambda^8\int_{Q_\omega}e^{-2s\alpha}\phi^4|\mathcal{V}|^2dtdx \\ &\quad +\frac{C}{\delta_2}s^7\lambda^9\int_{Q_\omega}e^{-2s\alpha}\phi^4|\mathcal{V}|^2dtdx \\ &\leq \delta_2\widetilde{\mathcal{I}}(\mathcal{U})+\frac{C}{\delta_2}s^7\lambda^9\int_{Q_\omega}e^{-2s\alpha}\phi^4|\mathcal{V}|^2dtdx\end{aligned}$$

Provided A > C and for S_2 > 0. Estimating the integral /C$_3$, we also get /C$_3$ <

$$\begin{aligned}\mathcal{K}_3 &\leq \delta_3 s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}|\mathcal{U}|^2dtdx+\frac{C}{\delta_3}s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}|B|^2|\mathcal{V}|^2dtdx \\ &\leq \delta_3\widetilde{\mathcal{I}}(\mathcal{U})+\frac{1}{\delta_3}s^7\lambda^9\int_{Q_\omega}e^{-2s\alpha}\phi^4|\mathcal{V}|^2dtdx,\end{aligned}$$

For sufficiently large $\lambda\geq C$ and for $\delta_2>0$ and for $\delta_3>0$. Again using Green's theorem, we have

$$\begin{aligned}\mathcal{K}_4 &\leq \delta_4 s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}|\mathcal{U}|^2dtdx+\delta_4 s\lambda^2\int_{Q_\omega}e^{-2s\alpha}\phi^{-2}|\nabla\mathcal{U}|^2dtdx \\ &\quad +\frac{C}{\delta_4}s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}|\mathcal{V}|^2dtdx+\frac{C}{\delta_4}s^5\lambda^6\int_{Q_\omega}e^{-2s\alpha}\phi^2|\mathcal{V}|^2dtdx \\ &\leq \delta_4\widetilde{\mathcal{I}}(\mathcal{U})+\frac{1}{\delta_4}s^7\lambda^9\int_{Q_\omega}e^{-2s\alpha}\phi^4|\mathcal{V}|^2dtdx,\end{aligned}$$

For any s > CT2 and A > 1 and for $S4$ > 0 Now we estimate the integral /C5 as follows

$$\begin{aligned}\mathcal{K}_5 &= s^3\lambda^4\int_{Q_\omega}e^{-2s\alpha}\mathcal{R}\rho\mathcal{U}\left[B_t z+D_t\nabla z+E_t y\right]dtdx \\ &= \mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3.\end{aligned}$$

It is easy to see from Lemma 1.2.1 that

$$\begin{aligned}\mathcal{L}_1 &\leq \delta_5 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}|\mathcal{U}|^2 dtdx + \frac{C}{\delta_5} s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}|B_t z|^2 dtdx \\ &\leq \delta_5 \widetilde{\mathcal{I}}(\mathcal{U}) + \frac{C}{\delta_5} s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^4|\mathcal{V}|^2 dtdx,\end{aligned}$$

Provided $s > CT2$ and A > 1. Now the estimate for the integral £2

$$\begin{aligned}\mathcal{L}_2 &\leq \delta_6 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}|\mathcal{U}|^2 dtdx + \delta_6 s\lambda^2 \int_{Q_\omega} e^{-2s\alpha}\phi^{-2}|\nabla\mathcal{U}|^2 dtdx \\ &+ \frac{C}{\delta_6} s^5\lambda^6 \int_{Q_\omega} e^{-2s\alpha}\phi^2 \left|\int_\theta^t \mathcal{V}(x,\tau)d\tau\right|^2 dtdx.\end{aligned}$$

In this assessment, the third integral can be estimated further with the help of the inequality between Cauchy and Swarz and Fubini theorem.

$$\begin{aligned}\int_{Q_\omega} e^{-2s\alpha}\phi^2\left|\int_\theta^t \mathcal{V}(x,\tau)d\tau\right|^2 dtdx &= \int_\omega\int_0^\theta e^{-2s\alpha}\phi^2\left|\int_t^\theta \mathcal{V}(x,\tau)d\tau\right|^2 dtdx \\ &+ \int_\omega\int_\theta^T e^{-2s\alpha}\phi^2\left|\int_\theta^t \mathcal{V}(x,\tau)d\tau\right|^2 dtdx \\ &= \mathcal{N}_1 + \mathcal{N}_2,\end{aligned}$$

Where $\theta = T/2$. the estimation of the integral $\mathcal{N}_1$) as follows

$$\begin{aligned}\mathcal{N}_2 &\leq CT\int_\omega\int_\theta^T \left(\int_\tau^T e^{-2s\alpha}\phi^2 dt\right)|\mathcal{V}|^2 d\tau dx \\ &\leq CT^2\int_\omega\int_\theta^T e^{-2s\alpha(x,\tau)}\phi^2(x,\tau)|\mathcal{V}(x,\tau)|^2 d\tau dx.\end{aligned}$$

Likewise the following estimate can be easily obtained JV2, *t* € *(9, T).*

$$\begin{aligned}\mathcal{N}_2 &\leq CT\int_\omega\int_\theta^T \left(\int_\tau^T e^{-2s\alpha}\phi^2 dt\right)|\mathcal{V}|^2 d\tau dx \\ &\leq CT^2\int_\omega\int_\theta^T e^{-2s\alpha(x,\tau)}\phi^2(x,\tau)|\mathcal{V}(x,\tau)|^2 d\tau dx.\end{aligned}$$

The two inequalities above followed from the second theorem of the mean value and the manner in which the weight function increases in the interval (0, 9) and decreases in (9, T) (both as for time only). Currently, we achieve the above two inequalities.

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