Multiplication product rule of THAKKURA PHERU and its generalized rule

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***Abstract*- The paper is addressed to the work of the mathematician Thakkura Pheru for obtain a product multiplication consisting of the same digit repeated nine times , where the digit is positive and less than 10. In the present study we generalized the rule which was given by the Indian mathematician Thakkura Pheru.**

Key Words: **Vedic Mathematics, Thakkura pheru, Ganitasarakaumudi (GSK), Multiplication.**

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INTRODUCTION: Thakkura Pheru is a great north Indian Mathematician. He was an author of books on Mathematics, coins and gems. Thakkura Pheru gave a rule from his book Ganitasarakaumudi (GSK-4.61) for obtaining a product of multiplication consisting of the same digit repeated nine times.

Viz;

$12345679 \times a \times 9 = aaaaaaaaa$

Where, 'a' is a positive integer and less than 10.

Example – 1. $12345679 \times 7 \times 9 = ?$

Answer: 777777777 ($\therefore$ Nine times 7 by Thakkura Pheru rule)

Example – 2. $12345679 \times 8 \times 9 = ?$

Answer: 888888888 ($\therefore$ Nine times 8 by Thakkura Pheru rule)

Here, we shall try to make a generalized rule for multiplication consisting of the same digit repeated nine times by giving suitable cases.

Case – I: When a = 0 and a $\in$ negative integer

Then, we get

$12345679 \times 0 \times 9 = 000000000$ ($\therefore$ Nine times 0)

Now if, a $\in$ negative integer

Then, we get

$12345679 \times (-a) \times 9 = -\{aaaaaaaaa\}$

Example – 3. Let a = -6

Then, we get

$12345679 \times (-6) \times 9 = -666666666$ ($\therefore$ 9 times 6 with negative sign)

Now the generalized rule –

$$12345679 \times a \times 9 = \begin{cases} aaaaaaaaa, & if \quad 0 \le a < 10, \quad a \in positive \quad \text{int } eger \\ -\{aaaaaaaaa\} & \\ If \quad -10 < a \le 0, \quad a \in negative \quad \text{int } eger \end{cases}$$

Case – II: When a $\in$ integer which is greater and equal to 10.

i.e., $a \geq 10, \quad a \in \textit{positive} \quad \text{int}\,\textit{eger}$

$$\left\{\begin{array}{ll} a=10, & 12345679\times 10\times 9=1111111110 \\ a=11, & 12345679\times 11\times 9=1222222221 \\ a=12, & 12345679\times 12\times 9=1333333332 \\ a=13, & 12345679\times 13\times 9=1444444443 \\ & \vdots \\ & \vdots \\ a=18, & 12345679\times 18\times 9=1999999999 \end{array}\right\}$$

If we take a = 19, than above rule doesn't work because the sum of digits "a" i.e., (19) must be less than 10. Here,

1 + 9 = 10 ; this is a contradiction.

Now , we see from 20 to 27, we get

$$\left\{\begin{array}{ll} a=20, & 12345679\times 20\times 9=2222222220 \\ a=21, & 12345679\times 21\times 9=2333333331 \\ a=22, & 12345679\times 22\times 9=2444444442 \\ a=23, & 12345679\times 23\times 9=2555555553 \\ & \textit{and so on} \\ a=27, & 12345679\times 27\times 9=2999999997 \end{array}\right\}$$

If we take a = 28 and a = 29 than above rule doesn't work because the sum of digits "a" (i.e., 28 and 29) must be less than 10.

Here,

2 + 8 = 10 and 2 + 9 = 11 ; this is a contradiction.

And so on

Now we establish generalized rule as below-

If a = a_1 , a_2 , where a_1 = 1st term of 'a' and a_2 = 2nd term of 'a'.

Than

$$12345679\times a\times 9=\left\{\begin{array}{l} a_1(a_1+a_2).....(8\ \textit{times})a_2 \\ \textit{if}\ \ (a_1+a_2)\leq 9 \end{array}\right\}$$

i.e.,

$$12345679\times a\times 9=\left\{\begin{array}{ll} a_1(a_1+a_2).....(8\ \textit{times})a_2 & 10\leq a\leq 18 \\ & 20\leq a\leq 27 \\ & 30\leq a\leq 36 \\ & \textit{and}\quad \textit{so on} \\ & a=90 \end{array}\right\}$$

Case – III: **When a $\in$ integer which is less and equal to 10.**

i.e., $a \leq -10, \quad a \in \textit{nagetive} \quad \text{int}\,\textit{eger}$

Example – 4. $12345679\times(-21)\times 9=-2333333331$

Example – 5. $12345679\times(-34)\times 9=-377777774$

If a = - (a_1 , a_2), where a_1 = 1st term of 'a' and a_2 = 2nd term of 'a', (a_1 , $a_2 \in Z^+$).

Than

we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \left\{ \begin{matrix} -a_1(a_1 + a_2).....(8\ times)a_2 \\ if\ (a_1 + a_2) \le 9 \end{matrix} \right\}$$

i.e.,

$$12345679 \times a \times 9 = \left\{ \begin{matrix} -\{\ a_1(a_1 + a_2).....(8\ times)a_2\}, & a \in [-10,\ -18] \\ & a \in [-20,\ -27] \\ & a \in [-30,\ -36] \\ & \\ & and\ \ so\ on \\ & a = -90 \end{matrix} \right\}$$

Case – IV: **When** $a \in$ *positive* int *eger* **; here a =a1, a2 and** $(a_1 + a_2) \ge 10$

$$\left(12345679 \times 19 \times 9 = 2111111109 \right)$$

$$\left(\begin{matrix} 12345679 \times 28 \times 9 = 3111111108 \\ 12345679 \times 29 \times 9 = 3222222219 \end{matrix} \right)$$

$$\left(\begin{matrix} 12345679 \times 37 \times 9 = 4111111107 \\ 12345679 \times 38 \times 9 = 4222222218 \\ 12345679 \times 39 \times 9 = 4333333329 \end{matrix} \right)$$

$$\left(\begin{matrix} 12345679 \times 46 \times 9 = 5111111106 \\ 12345679 \times 47 \times 9 = 5222222217 \\ 12345679 \times 48 \times 9 = 5333333328 \\ 12345679 \times 49 \times 9 = 5444444439 \end{matrix} \right)$$

$$\left(\begin{matrix} 12345679 \times 55 \times 9 = 6111111105 \\ 12345679 \times 56 \times 9 = 6222222216 \\ 12345679 \times 57 \times 9 = 6333333327 \\ 12345679 \times 58 \times 9 = 6444444438 \\ 12345679 \times 59 \times 9 = 6555555549 \end{matrix} \right)$$

And so on
Then, we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \left(\begin{matrix} (a_1 + 1)(m_1 + m_2).....,\ 7\ times\ m_2\ a_2 \\ where,\ \ m_1 = 1^{st}\ term\ of\ (a_1 + a_2) \\ m_2 = last\ term\ of\ (a_1 + a_2) \end{matrix} \right)$$

Example – 6. $12345679 \times 92 \times 9 = 1022222221\ 2$

Example – 7. $12345679 \times 99 \times 9 = 1099999998\ 9$

Case – V: **When a** $\in$ **negative integer**

$$such\ \ that \quad (a_1 + a_2) \ge 10\ ;\ \ where,\ \ a = -(a_1.a_2)$$
$$and\ a_1, a_2 \in +ve\ Z$$

Example – 8. $12345679 \times -(19) \times 9 = -2111111109$

And so on
Then, we establish generalized rule as bellow-

$$12345679 \times a \times 9 = \begin{pmatrix} -\{(a_1+1)(m_1+m_2).....,\ 7\ times\ m_2\ a_2\} \\ where,\ m_1 = 1^{st}\ term\ of\ (a_1+a_2) \\ m_2 = last\ term\ of\ (a_1+a_2) \end{pmatrix}$$

Example – 9. $12345679 \times -(55) \times 9 = ?$

Answer: here, a = -55

$a_1 = 5, a_2 = 5$

$(a_1 + a_2) \geq 10$

$m_1 = 1, m_2 = 0$

Required answer is

$= -\{\ (5=1)(1+0)(1+0)(1+0)(1+0)(1+0)(1+0)(1+0)\ 05\ \}$

$= -\ (\ 6111111105\ ).$

CONCLUSION

We have framed the generalized rule for different intervals. Hence the multiplication becomes easier when we follow the above rule.

REFERENCES:

[1] Thakura Pheru Ganitasarakaumudi: The moonlight of the Essence of mathematics, editaed with introduction, Translation and Mathematical commentary by Sakhya, Manohar, New Delhi, 2009, pp. xxi-xxii.

[2] D.B.Ojha, "A new property of multiplication magic square" Journal of Emerging Technologies and Innovative Research (JETIR), vol. – 6, No – 2, PP. 892-898, February 2009.

[3] B.S.Yadav and M.Mohan (eds.), Ancient Indian Leaps into Mathematics, published by Birkhasur, New York-2011.

[4] Sreeramula Rajeswar Sarma "Mathematical Literature in the Regionel Languages of India", Indian Journal of History of Science, 28, 40227 (2009), PP. 201-212.

[5] Sreeramula Rajeswara Sarma, "Some Medieval Airthmetical Tables", Indian Journal of History of Science, 32, (1997), 191-198.

[6] Sreeramula Rajeswara Sarma, "Mathematical Literature in Telgu ; An Overview", Sri Venkateswara University Oriental Journal, 28, 7790, (1985).