SPONTANEOUS ELECTRON-POSITRON PAIR CREATION FROM THE VACUUM

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Abstract:
In this paper it is found that a nucleus of atomic number Z=174 is capable of spontaneous electron positron pair creation. Gershtein and Zel'dovich [1], showed that momentum P is infinity at r=o ; it is opposed in this paper. Using special relativity, a relation to calculate ionisation energy (I.E.) for the final K electron is derived that closely agrees with the data of Lide [2].

Keywords: Pair production, spontaneous transition of electron, ionization energy.

INTRODUCTION:

In the paper [3] the possibility of pair production in the sun is discussed. The paper indicates that on the surface of the earth energy of a Fe nucleus is larger than the free space. The paper also indicates that if ϕ_{ES} is the gravitational potential on the surface of the earth, then the Fe nucleus must have kinetic energy of the order of $(\frac{1}{1+\frac{\phi_{ES}}{c^2}} - 1)m_{Fe}c^{\,2}$ to escape the earth, where m_{Fe} is the mass of the Fe nucleus. The Fe nucleus would be more energetic at the centre of the earth in comparison to the surface, but the energy is too low for pair production. To understand pair production study of spontaneous pair creation from vacuum is extremely important. Dirac did mathematically establish that spontaneous transition of an electron (of rest energy mc^2) from a positive energy state to a negative energy state lead to energy surplus of 2 mc^2 [4]. In the stimulated case, an electron of energy $2mc^2$ can create an electron-positron pair near a nucleus; that is the energy required to supply the electron is mc^2. Due to the electrostatic pull of the nucleus, the required energy to be supplied to the electron for vacuum decay is less near a nucleus of high value of atomic number Z (see page-4 of ref. [5]). According to Oppenheimer and Plesset [6], as the atomic number Z increases the energy required to supply the electron for vacuum decay decreases, subtracting rest mass one will get the required energy for an electron is $mc^2\left(1+\alpha^2Z^2/(1-\alpha^2Z^2)^{\frac{1}{2}}\right)^{-\frac{1}{2}}$,it is illustrated in figure 1, where we see that at Z=137, required energy is zero. Thus, Supercritical charge for spontaneous vacuum decay is Z_{cr} =137.

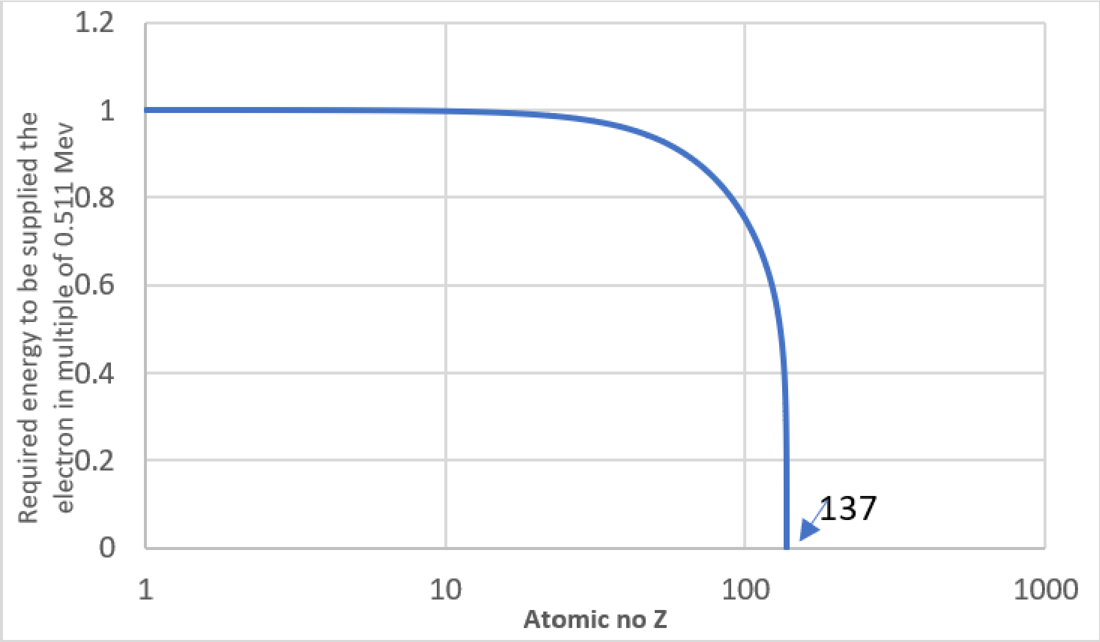

Figure 1. Required energy to be supplied to the electron for vacuum decay versus atomic number Z

RESULTS AND DISCUSSION:

Gershtein and Zel'dovich showed that momentum P is infinity at r=o using the relations $P = \frac{\hbar}{r}$ and $P = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$; According to them, v⟶c when $\frac{Ze^2}{\hbar c} \longrightarrow 1$, consequently momentum becomes infinity, it gives r=0 from the Bohr's relation $P = \frac{\hbar}{r}$. Therefore, they argued $\frac{Ze^2}{\hbar c} \longrightarrow 1$ corresponds to "collapse into the centre"[1].

To oppose this argument, let us start with the Bohr's relation $\frac{Pv}{r} = \frac{Ze^2}{r^2}$ mentioned in the aforesaid paper

As v⟶c, replacing P by γmv we get $\frac{\gamma mv^2}{r} = \frac{Ze^2}{r^2}$

$$\Rightarrow \gamma mv^2 = \frac{Ze^2}{r}$$

$$\Longrightarrow v = c \quad \text{if } r = \frac{Z}{\gamma}\, r_e \qquad \text{where } r_e = \frac{e^2}{mc^2}$$

So, at $v = c$ or γ is infinity, so r only appears as zero for an observer at the rest frame.

With respect to an observer in the rest frame one will get

$$\gamma = \sqrt{1 + Z^2\alpha^2} \qquad (1)$$

and

$$v = \frac{Z\alpha c}{\sqrt{1+Z^2\alpha^2}} \qquad (2)$$

where, $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$

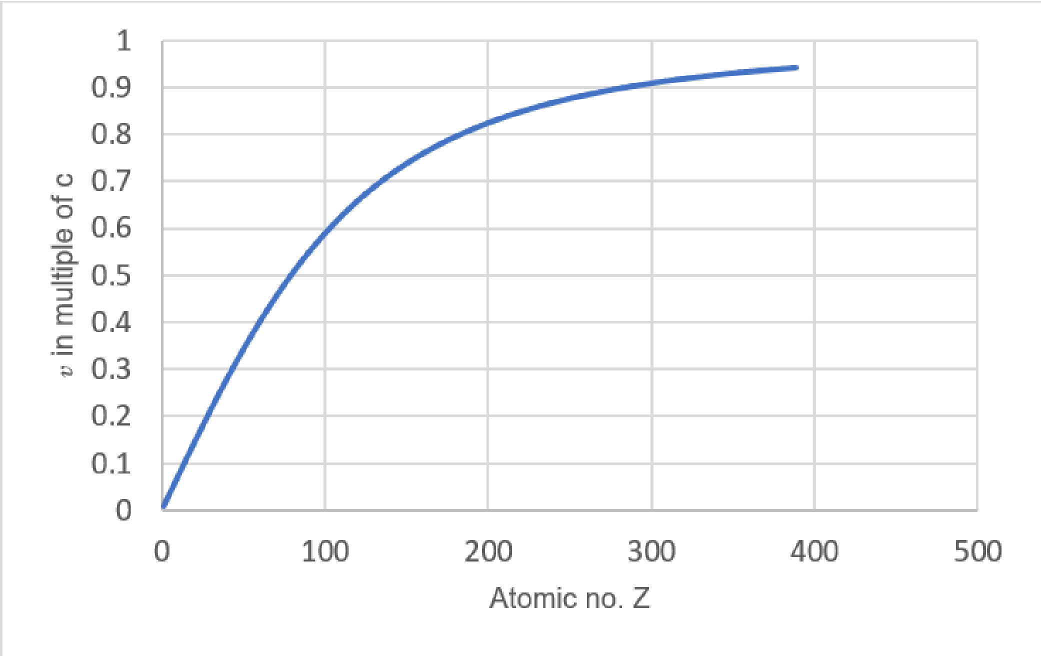

Figure 2. v versus Z graph from eq. (2)

Figure 2, illustrated that v never reaches c at any value of Z, so momentum P never reaches infinity. The interpretation "collapse into the centre" is the result of misunderstanding of special relativity.

The energy of the lowest atomic state

$$E = \gamma mc^2 - \frac{Ze^2}{r}$$

$$= \gamma mc^2 - Z\alpha Pc \qquad (since,\ \frac{Ze^2}{r} = \frac{Ze^2}{Pc\,r}Pc = \frac{Ze^2}{\hbar c}Pc = Z\alpha Pc) \qquad (3)$$

$$\Rightarrow E = mc^2\sqrt{1+Z^2\alpha^2} - Z\alpha\sqrt{\in^2 - m^2c^4} \qquad (\text{ when Z}\rightarrow 137,\ using\ eq.\ (1))$$

$$= mc^2\sqrt{1+Z^2\alpha^2} - Z^2\alpha^2 mc^2 \qquad (4)$$

Figure 3 shows the E versus Z graph using equation (4). From the figure 3 one may find that interface of positive and negative energy appears at Z=174.

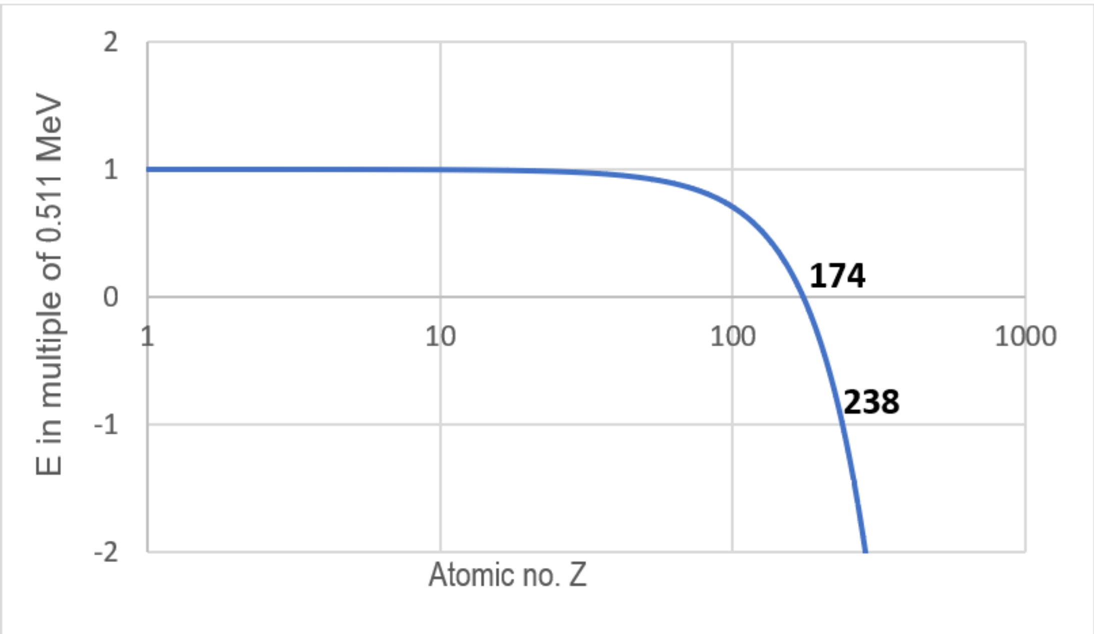

Figure 3. E versus Z graph using equation (4)

In the equation (3), If we substitute Pc =∈ (that is sometimes used for an electron if $v \to c$ (see page no. 315 of ref. [7])) instead of Pc $= \sqrt{\in^2 - m^2c^4}$, we will get $E = \sqrt{1+Z^2\alpha^2}(1 - Z\alpha)mc^2$, graph depicted in Figure 4, which shows E enters into the negative region at a critical charge exactly equal to137 i.e. $\frac{ze^2}{\hbar c} = 1$, it declines the criticism of Gershtein and Zel'dovich [1]. The difference is that for the graph plotted in the figure 4 we substitute Pc =∈ i.e. the rest mass of the electron is zero while the figure 3 is based on non-zero rest mass of an electron.

It can also be seen from equation (1) that, at another critical charge of Z=238, γ=2 and the Figure 3 gives that at Z=238, E=$-0.511MeV = -mc^2$; so the energy gap with Z=174 is mc^2. A gamma ray of energy only mc^2 can produce a pair in case of a nucleus of atomic number Z=174, while for an electron, its rest energy is sufficient to produce a pair (i.e. energy required to supply the electron is zero). One may compare it with Dirac's prediction of spontaneous transition of an electron to negative energy state.

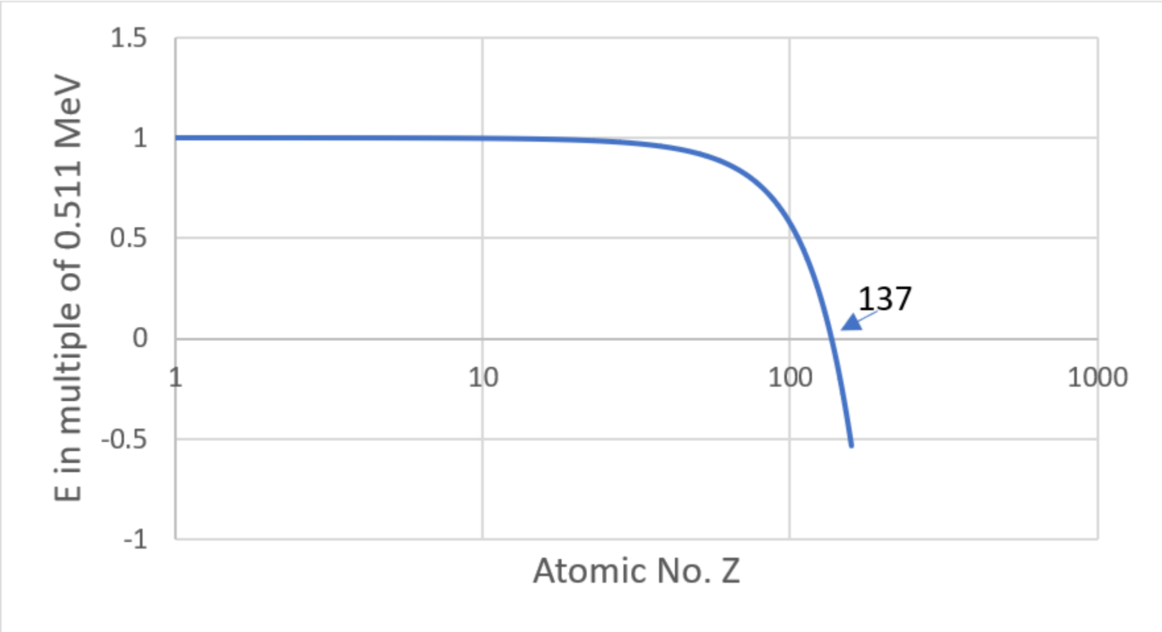

Figure 4. E versus Z graph

The reliability of special relativity at a high value of v is undeniable, which happens near a high value of Z, but is it equally fit for low value of Z whose I.E. are known? For low atomic number nuclei, the ionisation energy (I.E.) for the final K electron is found as

$$\text{I.E.} = (\gamma^2 - \gamma)\, mc^2 = (Z^2\alpha^2 - \sqrt{1+Z^2\alpha^2} + 1)mc^2 \qquad (5)$$

I.E. that are calculated with eq. (5) and from the reference Lide D. R. 2003 are shown in Table1 and are plotted in the Figure5. The two graphs overlap each other and can hardly be distinguished, which verifies the correctness of both eq. (5) and eq. (1).

Table- 1. Comparison of I.E calculated from equation (5) and from Lide [2]

Atomic No.	I.E. from Lide [2]	Equation (A26)		Atomic No.	I.E. from Lide [2]	Equation (A26)
1	13.6	13.61303		16	3494.19	3496.691
2	54.42	54.45428		17	3946.296	3949.141
3	122.45	122.5303		18	4426.23	4429.434
4	217.72	217.8519		19	4934.05	4937.643
5	340.23	340.4344		20	5469.86	5473.845

6	489.99	490.2972	21	6033.71	6038.12
7	667.05	667.4642	22	6625.82	6630.554
8	871.41	871.9635	23	7246.12	7251.234
9	1103.12	1103.828	24	7894.81	7900.251
10	1362.2	1363.093	25	8571.94	8577.702
11	1648.7	1649.801	26	9277.69	9283.684
12	1962.67	1963.995	27	10012.12	10018.3
13	2304.14	2305.726	28	10775.4	10781.65
14	2673.18	2675.047	29	11567.62	11573.85
15	3069.84	3072.015		Not mentioned	

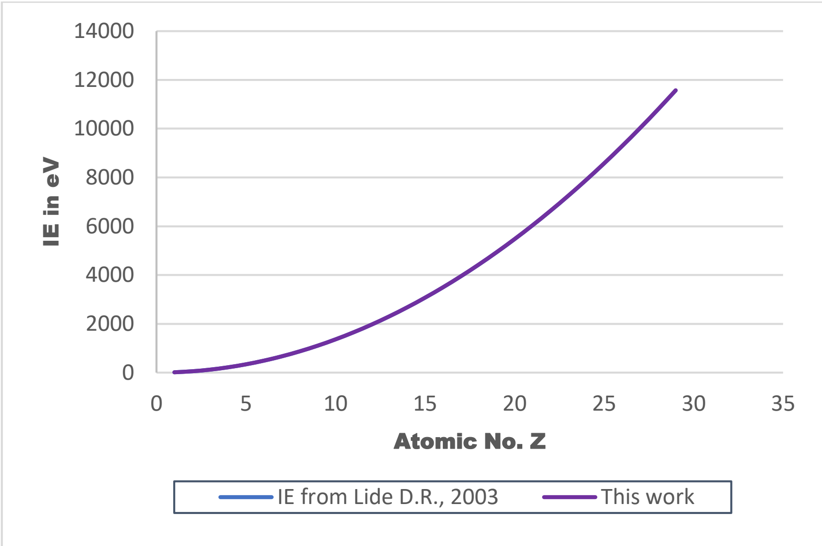

Figure 5. Graphical comparison of I.E calculated from equation (5) and from Lide[2]

CONCLUSION:

It is well known that $\boldsymbol{e^+ + e^- \leftrightharpoons quark + antiquark}$ **,** thus the study of pair production not only help us to extract energy from vacuum also help to understand creation of the universe. The mystery of matter antimatter symmetry may lie within it.

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