RECONSTRUCTION OF TWO TIME-INDEPENDENT COEFFICIENTS IN THE PHASE FIELD SYSTEM

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Abstract:
Conversely, in many branches of the applied math the value of certain model parameters is often obtained from internal measurements or border measurements. The principal goal of the paper is to determine stability results by establishing two time-independent coefficients for combined parabolic and Dirichlet equations resulting from phase changes. Phase field systems as mathematical models for phase transitions have been an important focus over recent years.

Key Words: Optimization, Measurements, Simultaneous, Reconstruction.

Introduction

From a thermodynamically point of view, we recall the extension of Stefan Enthalpic Modeling from the first introduced and new-improved field models, first introduced by several authors (see Penrose & Five's work, which gives an extensive account of underlying physics). Reconstructing two time-independent coefficients in a phase field system is an inverse problem that typically involves transforming it into an optimization problem. Techniques include using Carleman estimates to prove stability and uniqueness, or transforming the problem into an optimization problem and using methods like the gradient descent method or conjugate gradient method to find the solution. The process requires making assumptions about the model and using measurements of the solution at a specific time or over the entire domain. Several variants of the model, and interesting findings for the presence, regularity and dependence on the physical parameters were achieved. A parabolic equation system describes the transformation phases between two states (solid or fluid for example), in pure material (u, V) as solution, is the easiest linearization of the phase field models.

$$\left.\begin{array}{ll} u_t + l(x)v_t - \Delta u + a(x)v = f_1(x), & \text{in } Q, \\ v_t - \Delta v + b(x,t)v + c(x)u = f_2(x), & \text{in } Q, \\ u(x,\theta) = u_\theta(x),\ v(x,\theta) = v_\theta(x), & \text{in } \Omega, \\ u(x,t) = h_1(x,t),\ v(x,t) = h_2(x,t), & \text{on } \Sigma, \end{array}\right\} \tag{5.48}$$

Limit 30 of Class C2. The coefficient l € L °(0) is the latent warmth, b £ Cl(Q) and the semi-initial values ug for some fixed values of 9€ (0, T) and vg: 0 - >• R are regular (i.e., (ug Vg) £ (H2(O))))2). S-s>r shall be kept fixed and the function (/i, /2) shall be given (t2(o))2. Not a null smooth Dirichlet. The u-solution suggests that in both phases and the smooth function a material with a heat distribution of region 0 can be solid or liquid (when the temperature of melting is zero) v is called 'phase field function' so that one phase, for example liquid and v near the other solid phase, is corresponding to v near +1. v is called phase field function. The phase field function can be connected and distinguished by the macroscopic monitors. In many fields of statistical mechanics it was also associated with microscopic quantities. The order parameter and other variables are connected to each other in a system of complex dynamics and there is a fixed value restricting the balance curve in the pressure-temperature level. Unknown coefficients a(x) and c(x) are considered to be sufficiently flat and are kept separate from time t. The objective of this chapter is to obtain a Lipschitz stability estimate for the determination of the coefficients a(x) and c(x) via internal measurement of one observation in a limited field. The key ingredient for these stability findings is an L2-weighted inequality for solutions for phase field systems, explained briefly below. The controllability of the phase field system[6] has been investigated by Barbu. When we obtain an estimate from Carleman with two observations, we follow the method used in [6] with various transformations related to reverse problems and infer a new estimate from Carleman using certain energy type estimates with one observation. The initial reconstruction in a heat-conductor system was simultaneously investigated for temperature and heat radiation coefficient.

With regard to the stability estimate, one (or two) reverse parabolic systems with variable and constant coefficients were recently used in the worldwide Carleman estimates for the measurement of the reverse. This includes the simultaneous reconstruction, with a solution above (f0, T) x fi, of one diffusion system coefficient and initial conditions (t0,T). Reversely, all or some coefficients of the reaction-diffusion-convection-system were investigated through observations in an arbitrary sub-domain over a time-interval of only one and two components in a fixed positives time 0. We wish to determine the coefficients a and c by side data of only one component. The description is as follows. The system solutions for coefficients (l, a, b, c) and the same border data with the form semiconductor should (u, v) be the following {ue,v$)

$$\left.\begin{aligned} &\widetilde{u}_t + l(x)\widetilde{v}_t - \Delta\widetilde{u} + \widetilde{a}(x)\widetilde{v} = f_1(x), && \text{in } Q,\\ &\widetilde{v}_t - \Delta\widetilde{v} + b(x,t)\widetilde{v} + \widetilde{c}(x)\widetilde{u} = f_2(x), && \text{in } Q,\\ &\widetilde{u}(x,\theta) = \widetilde{u}_\theta(x),\ \widetilde{v}(x,\theta) = \widetilde{v}_\theta(x), && \text{in } \Omega,\\ &\widetilde{u}(x,t) = h_1(x,t),\ \widetilde{v}(x,t) = h_2(x,t), && \text{on } \Sigma. \end{aligned}\right\} \tag{5.49}$$

We set p(x, t) = u(x, t) — u(x, t), q(x, t) = v(x, t) — v(x, t), f(x) = a(x) — a(x) and g(x) = c(x) — c(x) so that the subtraction of from yields

$$\left.\begin{aligned} &p_t + l(x)q_t - \Delta p + a(x)q = fR, && \text{in } Q,\\ &q_t - \Delta q + b(x,t)q + c(x)p = g\mathcal{R}, && \text{in } Q,\\ &p(x,\theta) = p_\theta,\ q(x,\theta) = q_\theta, && \text{in } \Omega,\\ &p(x,t) = q(x,t) = 0, && \text{on } \Sigma, \end{aligned}\right\} \tag{5.50}$$

Where $(x,t) = -\widetilde{v}(x,t)$ and $\mathcal{R}(x,t) = -\widetilde{u}(x,t)$. Here and henceforth, for our convenience, we denote ((x, 0) := Co- Again by a simple computation with the transformations the system) becomes

$$y(x,t) = p(x,t) \quad \text{and} \quad z(x,t) = \frac{q(x,t)}{\mathcal{R}(x,t)}, \quad \text{for all } (x,t) \in Q,$$

$$\left.\begin{array}{ll} y_t + Kz_t - \Delta y + Az = fR, & \text{in } Q, \\ z_t - \Delta z + Bz + D\nabla z + Ey = g, & \text{in } Q, \\ y(x,\theta) = p_\theta, \quad z(x,\theta) = \dfrac{q_\theta}{\mathcal{R}_\theta}, & \text{in } \Omega, \\ y(x,t) = z(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad (5.51)$$

Where the coefficients

$$K(x,t) = l\mathcal{R}, \quad A(x,t) = a\mathcal{R} + l\mathcal{R}_t, \quad B(x,t) = \frac{\mathcal{R}_t}{\mathcal{R}} - \frac{\Delta\mathcal{R}}{\mathcal{R}} + b,$$

$$D(x,t) = -\frac{2\nabla\mathcal{R}}{\mathcal{R}} \quad \text{and} \quad E(x,t) = \frac{c}{\mathcal{R}}.$$

Further we set 14 = yt and V = zt; then (14, V) satisfies

$$\left.\begin{array}{ll} \mathcal{U}_t + K\mathcal{V}_t - \Delta\mathcal{U} + A\mathcal{V} + K_t\mathcal{V} + A_t z = fR_t, & \text{in } Q, \\ \mathcal{V}_t - \Delta\mathcal{V} + B\mathcal{V} + D\nabla\mathcal{V} + E\mathcal{U} + F^{yz} = 0, & \text{in } Q, \\ \mathcal{U}(x,\theta) = \mathcal{F}_\theta, \quad \mathcal{V}(x,\theta) = \mathcal{G}_\theta, & \text{in } \Omega, \\ \mathcal{U}(x,t) = \mathcal{V}(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad (5.52)$$

Where

$$F^{yz} = B_t z + D_t\nabla z + E_t y$$

and the semi-initial data

$$\mathcal{F}_\theta = fR_\theta - K_\theta\mathcal{G}_\theta + \Delta y_\theta - A_\theta z_\theta$$

$$\text{and} \quad \mathcal{G}_\theta = g + \Delta z_\theta - B_\theta z_\theta - D_\theta\nabla z_\theta - E_\theta y_\theta.$$

In order to proceed further, we make the following assumptions:

Assumption 5.1.1. Suppose the time-independent coefficient $c(x) \geq c_0 > 0$ in ω_2 exists, where $\omega_2 \Subset \omega \Subset \Omega.$

If the source term and the coefficients are found to meet certain conditions of smoothness and compatibility, the following limitations apply.:

Assumption 5.1.2. Suppose $|\tilde{u}|, |\nabla\tilde{u}|, |\Delta\tilde{u}|, |\nabla(\Delta\tilde{u})|, |\tilde{u}_t|, |\tilde{u}_{tt}|, |\nabla\tilde{u}_t|, |\Delta\tilde{u}_t|, |\tilde{v}|$ $|\tilde{v}_t|$ t\

are bounded by a positive constant $M_1 = M_1(\Omega, T) > 0$ *for all* $(x,t) \in Q.$

Hello and h2 of the Dirichlet boundary data are regular enough. Furthermore, we assume that the terms Pff, Apg, qg, Vqg and Aqg are given for certain fixed measures of zt in Qu and some fixed measures of 6 € (0,T). Now, the question under consideration is whether we can determine for cases -ug = ug, vg = vg and ug ^ ilg, vg / vg. the time-independent ratings a and c from the above measurements. In particular, the systems (6.1.1) and (6.1.2) should be solutions (u,v) and (u,v). Then there is a constant C > 0, according to Q,u,T,Mi and c0, which are sufficient for smooth and qg

$$\|a-\tilde{a}\|^2_{L^2(\Omega)}+\|c-\tilde{c}\|^2_{L^2(\Omega)}\le C\|q\|^2_{H^1(0,T;L^2(\omega))},$$

for the case $u(x,\theta)=\tilde{u}(x,\theta)$, $v(x,\theta)=\tilde{v}(x,\theta)$ and

$$\|a-\tilde{a}\|^2_{L^2(\Omega)}+\|c-\tilde{c}\|^2_{L^2(\Omega)}\le C\Big(\|q\|^2_{H^1(0,T;L^2(\omega))}+\int_\Omega(|q_\theta|^2+|\nabla q_\theta|^2+|\Delta q_\theta|^2+|p_\theta|^2+|\Delta p_\theta|^2)dx\Big),$$

First, we show a Carle man type estimate for the field phase system with only one observation on the right hand side of the subdomain u estimate of 0. Carle's assessment in Part 4 for a similar model with variable coefficients cannot be used specifically here, as it has proofed that the two coefficients have been identified on the left and right sides with differences in diffusion coefficient. The assessment is based on a single observation.

First, we define two weight functions that are helpful in the follow-up. Let u) be a fixed 0 arbitrary subdomain. First, we assume that a regular and positive function can be found that fulfills certain properties

$$\left.\begin{array}{l}\psi(x)>0\ \ \forall x\in\Omega,\quad \psi(x)=0\ \ \forall x\in\partial\Omega,\\ |\nabla\psi(x)|>0\ \forall x\in\bar{\Omega}\backslash\omega_0 \text{ and } \frac{\partial\psi}{\partial\nu}\le 0\ \forall x\in\partial\Omega,\end{array}\right\} \quad (5.53)$$

Where v indicates the normal outward to 30 for this function, one could refer to it. The weight functions are now introduced p, a: Q —> E with A > 1, t € (0, T):

$$\phi(x,t)=e^{\lambda\psi(x)}/\beta(t)\ \text{ and }\ \alpha(x,t)=\left(e^{2\lambda\|\psi\|_{C(\bar{\Omega})}}-e^{\lambda\psi(x)}\right)/\beta(t), \quad (5.54)$$

What @ (t) = t (T -1). Please note the weight function an is positive, blowing to +oo at t = 0 and t = T. As a result, e~2sa, pe~2sa, etc. functions are smooth and disappear in t = 0 and t = T. Note that for all (x, t) > C > 0 € Q and < C < oo all for all e > 0 and R for me. Moreover, we need the following assessments for functions p and an in order to prove the main inequalities. Here and now, we will describe a general positive constant with C(S2) whose value varies from line to line with ip and its derivatives and Q,T.

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