Using an economic production quantity model for decaying objects, the three-stage system has a partial backlog

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Abstaract:
This study focuses on the problem of deteriorating EOQ models for decaying things under three different scenarios. Partial backlogs and shortages are allowed under the proposed paradigm. We also show the time-convexity of the total cost function. The model is demonstrated with numerical examples. Finally, a sensitivity analysis is used to validate the proposed model. Mathematica software is used to find numerical solutions.

Keywords: Convexity, inventory, degradation, discount rate, and shortages.

INTRODUCTION:
Determining the optimal stock quantity and timing is essential to meeting future demand. The main objective of inventory management is to lower the cost of carrying inventory. Over the past few decades, a lot of research has been done on inventory problems with deteriorating products. Due to damage, evaporation, or other factors, some of the items are not in the best condition to satisfy the demand. Commodities that may adequately deteriorate during the duration of normal storage include food goods, cereals, fruits, vegetables, pharmaceuticals, radioactive materials, and electronic materials. Therefore, the loss caused by deterioration cannot be ignored. The effects of inflation and the time value of money were not considered in the majority of the traditional inventory models. It has been noted that the amount of stock level typically influences the demand rate in supermarkets; that is, the demand rate may fluctuate in tandem with the amount of stock on hand. Harris was the first to introduce the EOQ model [1]. The model had no degradation function and was predicated on steady demand. However, the demand may fluctuate over time in real life. By taking into account two distinct scenarios, Goel [2] developed on Economic Order Quantity under Conditions of Permissible Delay in Payments The effects of inflation were not considered in the majority of the traditional inventory models. Different outcomes were obtained when Chand and Ward [3] examined Goel's dilemma [2] using the presumptions of the classical economic order quantity model. An alternate method for calculating the economic order quantity under the circumstances of a permitted payment delay was created by Chung [4], [5]. For exponentially decaying products, Hwang and Shinn [6] designed an inventory system for the retailer's pricing and lot sizing policy, subject to allowable payment delays. The ideal period for payment within the allowable payment delay with degradation was discussed by Jamal et al. [7] and Sarker et al. [8]. The model with two parameters and a variable degradation rate was created by Covert and Philips [9]. The distribution of Weibull The inventory model was then created by Philip [10] using the Weibull distribution rate without shortage and three parameters. The majority of things in real life would have a shelf life of some time. There isn't any degradation taking place at that time.An inventory model for a non-instantaneous degrading item with stock-dependent demand was created by Wu et al. [11]. For degrading items in a falling market, Tripathy and Misra [12] created an inventory model that allows the shopkeeper to settle the account against the purchase he made when payment is delayed. In order to clear the account against the purchases made, Tripathy and Pradhan [13] created an inventory model for weibull degrading items with continuous demand while payment delays are permitted to the store.

In the study of inventory systems, shortages are a significant factor in certain economic operations. It is presumed that it is either totally lost or backlogged when shortages happen. However, in reality, some buyers are prepared to wait for backorders, while others would rather purchase from other vendors. Meheret al. [14] created an inventory model with a Weibull deterioration rate that accounts for payment delays in a market with diminishing demand. When shortages are permitted and there is a partial backlog, Uthayakumar and Geetha [15] created an inventory model for items with a finite planning horizon and non-instantaneous deterioration that has a stock-dependent demand rate An EOQ model for perishable goods with power demand and partial backlog was developed by Singh et al. [16] by taking into account the backlogging rate, which is inversely proportional to the waiting time. Time-dependent partial backlog and time-dependent deterioration were taken into account when Basu and Sinha [17] created a general inventory model. By taking into account shortages and shortages that are fully backlogged, Patra et al. [18] created a deterministic inventory model in situations where the rate of deterioration is time dependent and the demand rate is a nonlinear function of selling price. An inventory model was created by Tripathi [19] for goods with exponential demand rates and allowable payment delays in the event of shortages Scholars like Park [20], Hollier, and Mak [21] created inventions
partial backorder models for tory models. A model of manufacturing inventory with partially backlogged shortages was developed by Goyal and Giri [22]. Hou [23] created a model of inventories for products that are degrading and have a stock-dependent demand. He believed that the shortages were entirely backordered. Numerous studies on shortages, including those by Yang [24], Sana [25], Ye [26], Nasab [27], Konstantaras [28], and Wee [29], are noteworthy.

Like Donaldson, many researchers took time-varying demand into account [30]. Hariga [31]. Meal and Silver [32], etc.Both a constant and an exponential rate of deterioration were taken into consideration by Skouri and Papachristos [33] and Skouri et al. [34]. With time-dependent deterioration and partial backlog, Sana [35] extended the ideal selling price and lot size.

This is how the rest of the paper is structured. Section 2 presents assumptions and notations, whereas Section 3 presents mathematical formulations. Section 4 has a numerical example. Section 5 provides sensitivity analysis for a number of parameters. The final piece, piece 6, offers a conclusion and recommendations for the future.

ASSUMPTIONS AND NOTATIONS:

Throughout the work, the following presumptions are made.

(a) Demand rate of the items is constant.
(b) Replenishment rate is infinite and leadc time is zero.
(c) System operates for a prescribed period of planning horizon.
(d) Shortages are allowed only for a 'δ' fraction of it is backlogged. The remaining fraction (1-δ) is lost.
(e) Inventory constrains three steps in a cycle.
(f) Product has no deterioriation for fresh product time i.e. first step.
(g) Product transactions are followed by instantaneous cash flow.

In addition, the following notations are made all over the manuscript:

f	discount rate, representing the time value of money
i	inflation rate
$r = f - i$	net discount rate of inflation
H	planning horizon
T	replenishment cycle
m	number of replenishment during the planning horizon, m = H/T
T_d	length of time in whcih the product has no deteriorations (fresh product time)
Q	deteriorating rate
A	cost of replenishment $/order

I_m	maximum inventory level
I_b	maximum amount of shortage to be backlogged
p	per unit cost of items $/unit
h	per unit inventory holding cost / unit time, $/unit/unit time
b	per unit opportunity cost due to lost sales, $/unit
C_1	present unit value of ordering cost
H_1	present value of holding cost
s	per unit shortage cost/unit time, ($/unit/unit time)
S	present value of shortage cost
C_2	present unit value of opportunity cost
M	present value of material cost
$Z(m, k)$	present value of total cost
$T(m, k)$	present value of total cost of the system over a finite planning horizon
T_1	time at which backlog starts
$I_1(t)$	inventory level at time t, $0 \leq t \leq T_d$
$I_2(t)$	inventory level at time t, $T_d \leq t \leq T_1$
$I_3(t)$	inventory level at time t, $T_1 \leq t \leq T$.

MATHEMATICAL FORMULATIONS:

Let us consider the planning horizon H divided into m equal parts and length T = H/m. thus the order times over the planning horizon H are Tj = jT (J = 0, 1, 2, 3, ..., m). When the inventory is positive, demand rate is constant whereas for negative inventory, the demand is partially backlogged. The first replenishment lot size of I_m is replenished at T = 0. During the interval [0, T_d], the inventory level decreases due to the constant demand rate. The inventory level drops to zero due to demand and deterioration during [T_d, T_1]. During the interval [T_1, T], shortages occur and are accumulated unit t = T_1, before they are partially backlogged. The inventory system can be represented by the following equations:

$$\frac{dI_1(t)}{dt} = -D, \quad 0 \leq t \leq T_d \qquad (3.1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D \quad T_d \leq t \leq T_1 \qquad (3.2)$$

$$\frac{dI_3(t)}{dt} = -D\delta, \quad T_1 \leq t \leq T \qquad (3.3)$$

$$I_1(0) = I_m, \quad I_2(T_1) = 0 \quad I_3(T_1) = 0. \qquad (3.4)$$

The solution of the above differentiated equations after applying the boundary conditions (3.4) we obtain:

$$I_1(t) = I_m - D(t) \quad 0 \leq t \leq T_d \qquad (3.5)$$

$$I_2(t) = \frac{D}{\theta}\{e^{\theta(T_1-t)} - 1\}, \quad T_d \leq t \leq T_1 \qquad (3.6)$$

$$I_3(t) = -D\delta\{t - T_1\}, \quad T_1 \leq t \leq T \qquad (3.7)$$

Coordinating the continuities of I(t) at T = T_d it follows that

$I_1(T_d) = T_1(T_d)$which implies that (3.8)

$$T_2(I_m) = D\left\{T_d + \frac{1}{\theta}\left(e^{\theta(T_1-T_d)} - 1\right)\right\}$$

Substituting (3.8) into (3.5) we get

$$I_1(t) = D\left\{T_d + \frac{1}{\theta}\left(e^{\theta(T_1-T_d)} - 1\right)\right\} - Dt. \qquad (3.9)$$

Thus the maximum inventory level and maximum amount of shortage demand to be backlogged during the first replenishment cycle is :

$$I_m = D\left\{T_d + \frac{1}{\theta}\left(e^{\theta\left(\frac{kH}{m} - T_d\right)}\right)\right\} \quad (3.10)$$

$$I_b = D\delta\left(\frac{H}{m}\right)(1-k), \quad (3.11)$$

respectively.

There are m cycle during the planning horizon. Since inventory is assumed to start and end at zero, an extra replenishment at T_m = H is required to satisfy the backorders of the last cycle in the planning horizon. Therefore there are m + 1 replenishment in the entire planning horizon H.

$Q - I_m + I_b$ (3.12)

and the last of (m + 1)th Replenishment lost size is I_b.

The present value of ordering cost during the first cycle is

C_1 = A.

The present value of holding cost during the first replenishment cycle is

$$H_1 = h\left[\int_0^{T_d} I_1(t)e^{-rt}dt + \int_{T_d}^{T_1} I_2(t)e^{-rt}dt\right]$$

$$= hD\left[\left\{T_d + \frac{1}{\theta}\left(e^{\theta(T_1 - T_d)} - 1\right)\right\}\frac{(1 - e^{-rT_d})}{r} + \frac{T_d - e^{-rT_d}}{r} - \frac{1 - e^{-rT_d}}{r^2}\right]$$

$$+ \frac{hD}{\theta}\left[e^{\theta T_i} + \frac{e^{-rT_1} - e^{-rT_d}}{r}\right] \quad (3.14)$$

Shortages are partially backlogged. The present value of shortage cost during the first replenishment cycle is

$$S = -s\int_{T_1}^{T} I_3(t)e^{-rt}dt = sD\int_{T_1}^{T}(t - T_1)e^{-rt}\,dt$$

$$= \frac{D\delta}{r}\left[(T - T_1)e^{-rT} + \frac{e^{-rT} - e^{-rT_1}}{r}\right]$$

$$= \frac{sd\delta}{r^2}\left[\left\{\frac{rH}{m}(k-1) - 1\right\}e^{-\frac{rH}{m}} + e^{-\frac{rkH}{m}}\right]. \quad (3.15)$$

The present value of opportunity cost due to lost sales during the first replenishment cycle is :

$$C_2 = b\int_{T_1}^{T} D(1-\delta)e^{-rt}\,dt$$

$$= \frac{bD(1-\delta)}{r}\left(e^{-rT} - e^{-rT_1}\right)$$

$$= \frac{bD(1-\delta)}{r}\left(e^{-rkH/m} - e^{-rH/m}\right) \quad (3.16)$$

Replenishment is done at t = 0 and t = T. The present value of material cost during the first replenishment cycle is

$$M = pI_m + pIde^{-rT}$$

$$= pD\left\{T_d + \frac{1}{\theta}\left(e^{\theta\left(\frac{kH}{m} - T_d\right)} - 1\right)\right\} + pD\delta\left(\frac{H}{m}\right)(1-k)e^{-rH/m}. \quad (3.17)$$

The present value of the total cost of the system during a finite planning horizon H is :

$Z(m, k) = C_1 + H_1 + S + C_2 + M.$ (3.18)

The present value of total cost of the system over a finite planning horizon is:

$$T(m,k) = \sum_{i=0}^{m-1} Z(m,k)e^{-irT} + A.e^{-rH} = Z(m,k)\left(\frac{1-e^{-rH}}{1-e^{-rH/m}}\right) + A.e^{-rH}$$

Truncated Cityplace Taylor series is used for exponential terms to find the closed form solution i.e. $e^{-rT} \approx 1 - rT + \frac{r^2T^2}{2}$ (for finding closed form optimal soluion)

$$H_1 = hDT_d\left[\left\{T_1 + \frac{\theta(T_1-T_d)^2}{2}\right\}\left(1-\frac{rT_d}{2}\right) - \frac{T_d}{2}\left(1-\frac{rT_d}{2}\right)\right] + \frac{hD}{\theta}(T_1-T_d)\left[\theta T_1 + \frac{\theta^2T_1^2}{2} + \theta\left(\frac{\theta+r}{2}\right)T_1(T_1+T_d)(1+\theta T_1) + \frac{r}{2}(T_1+T_d)\right] \quad (3.19)$$

$$S = \frac{Ds\delta}{2}[(T^2-T_1^2) - T(rT^2 - rT.T_1 + 2T_1)] \quad (3.20)$$

$$C_2 = \frac{HbD(1-\delta)(1-k)}{m}\left\{1-\frac{rH}{2m}(1+k)\right\} \quad (3.21)$$

$$M = pD\left\{T_1 + \frac{\theta(T_1-T_d)^2}{2}\right\} + pD\delta(H/m)(1-k)\left(1-\frac{rH}{m}\right) \quad (3.22)$$

$$TC(m,k) = (C_1 + H_1 + S + C_2 + M)\left(\frac{1-e^{-rH}}{1-e^{-rH/m}}\right) + Ae^{-rH} \quad (3.23)$$

$$= (C_1 + H_1 + S + C_2 + M)G + Ae^{-rH}\left(T_1 = \frac{rH}{m}T = \frac{H}{m}\right).$$

For finding optimal solution taking partial derivative of TC (m, k) with respect to k and equating to zero we obtain $\frac{\partial TC(m,k)}{\partial k} = 0$.We obtain

$$\frac{HbD(1-\delta)}{m}\left(1-\frac{rHk}{m}\right) - \frac{Ds\delta H}{2m}(2T_1 + 2T - rT^2) + \frac{hDT_dH}{m}\left[\{1+\theta(T_1-T_d)\}\left(1-\frac{rT_d}{2}\right)\right] + \frac{hDH}{\theta m}(T_1-T_d)\left[\theta T_1 + \frac{\theta^2T_1^2}{2} + \theta\left(\frac{\theta+r}{2}\right)T_1.(T_1+T_d)(1+\theta T_1) + \frac{r}{2}(T_1+T_d)\right] + \frac{hDH}{\theta m}(T_1-T_d)\left[\theta + \theta^2T_1 + \theta\left(\frac{\theta+r}{2}\right).\{(2T_1 + T_d(1+\theta T_1) + \theta T_1(T_1+T_d)\} + \frac{r}{2}\right] + \frac{pDH}{m}\left[1 + \theta\left(\frac{kH}{m} - T_d\right) - \delta\left(1-\frac{rH}{m}\right)\right] = 0. \quad (3.24)$$

Taking second differential with respect to k we get $\frac{\partial^2 TC\,(m.k)}{\partial k^2} > 0$, which shows that the value of $T_1 = T_1^*$ and TC (m.k) = TC* (m, k) obtained from (3.23) and (3.24) is minimum.

NUMERICAL EXAMPLE:

Let us consider a = 250, h = 1.2, s = 2.2, b = 1.8, Q = 0.06, T_d = 0.08, δ= 0.5, H = 10, r = 0.2, D = 800, T = H/m in appropriate units. We obtain Optimal value of k = k* = 0.1, $T_1 = T_1^*$ = 0.0790479, optimal (minimum), Z = Z* = $1660.82.

SENSITIVITY ANALYSIS:

Taking the parameters as in above mentioned numerical example. The only variation is in the value of k.

Table 1: The variation of k keeping m = 2 (constant)

k	$T_1 = T_1^*$	TC (m, k) = TC* (m, k)	k	$T_1 = T_1^*$	TC (m, k) = TC* (m, k)
0.2	0.0790753	1354.81	0.7	0.079192	364.736
0.3	0.791012	1084.79	0.8	0.079212	274.724
0.4	0.0791258	850.77	0.9	0.079231	220.713
0.05	0.079149	652.761	1.0	0.0792491	202.702

Table 2 : The variation of m keeping k = 0.1 (constant)

m	$T_1 = T_1^*$	TC (m, k) = TC* (m, k)	m	$T_1 = T_1^*$	TC (m, k) = TC* (m, k)
2	0.790479	1660.82	7	0.784899	2447.59
3	0.788547	4019.84	8	0.0784395	2189.5
4	0.787225	3695.81	9	0.783965	1984.26
5	0.786254	3196.22	10	0.0783589	1818.28

From Table 1, weobserve that increase of fraction of scheduling constant time 'k' results increase inaptimal$T_1 = T_1^*$ and decrease in optimal total enat TC(m, k) = TC* (m, k).

From Table 2, we observe that the increase in the value of number of replenishmentduring the planning horizon m= H /T, results in decrease in the optimal time $T_1 = T_1^*$and TC (m, k) = TC*(m, k) increases from value m = 2 to m= 3 and again decreasesfrom m = 4 and onwards.

CONCLUSION:

This paper bas been developed for deterministic inventory model for deterioratingitems with constant demand rate over a finite planning horizon. Shortages are allowedand partially backlogged. We have also considered the effect of inflation in formulating theinventory policy. We have given the numerical formulation of the Problem to present anoptimal solution. Truncated CityplaceTaylor series is used for finding numerical optimalclosed form solution. Sensitivity analysis has been carried out with respect to variousparameters, the result shows that the effect of inflation on present value of the total costis more significant.

The paper can be extended in several ways, for instance we may extend the paper forexponential demand rate as well as quadratic time dependent demand rate. We couldalso generalize the model for time dependent holding cost etc.

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APPENDIX:

All partial derivatives with respect to k is given as :

$$\frac{\partial M}{\partial k}=\frac{pHD}{m}\left[1+\theta\left(\frac{kH}{m}-T_d\right)-\delta\left(1-\frac{rH}{m}\right)\right]\text{and}\frac{\partial^2 M}{\partial k^2}=\frac{\theta pDH^2}{m^2}$$

$$\frac{\partial C_2}{\partial k}=\frac{HbD(1-\delta)}{m}\left(1-\frac{rHk}{m}\right)\text{and}\frac{\partial^2 C_2}{\partial k^2}=-\frac{bHD(1-\delta)}{m^2}$$

$$\frac{\partial S}{\partial k}=\frac{Ds\delta H}{2m}(2T_1+2T-rT^2)\text{and}\frac{\partial^2 S}{\partial k^2}=-\frac{Ds\delta H^2}{m^2}$$

$$\frac{\partial H_1}{\partial k}=\frac{hDT_dH}{m}\left[\{1+\theta(T_1-T_d)\}\left(1-\frac{rT_d}{2}\right)\right]$$

$$+\frac{hDH}{\theta m}(T_1-T_d)\left[\theta T_1+\frac{\theta^2T_1^2}{2}+\theta\left(\frac{\theta+r}{2}\right)T_1.(T_1+T_d)(1+\theta T_1)+\frac{r}{2}(T_1+T_d)\right]$$

$$+\frac{hDH}{\theta m}(T_1-T_d)\left[\theta+\theta^2T_1+\theta\left(\frac{\theta+r}{2}\right)\{(2T_1+T_d(1+\theta T_1)+\theta T_1(T_1+T_d)\}+\frac{r}{2}\right]$$

$$\frac{\partial^2 H_1}{\partial k^2}=\frac{hDT_dH}{m}\left[\frac{\theta H}{m}\left(1-\frac{rT_d}{2}\right)\right]+\frac{hDH}{\theta m}\left[\frac{\theta H}{m}+\frac{\theta^2kH^2}{m^2}+\frac{\theta(\theta+r)}{2}\right.$$

$$\left.\left(\frac{2kH^2}{m^2}+\frac{HT_d}{m}\right)(1+\theta T_1)+T_1(T_1+T_d)\frac{\theta H}{m}+\frac{rH}{2m}\right]$$

$$+\frac{hDH}{\theta m}.\frac{H}{m}\left[\theta+\theta^2T_1+\frac{\theta(\theta+r)}{2}\left\{\frac{2H}{m}(1+\theta T_1)+(2T_1+T_d)\frac{\theta H}{m}+\right.\right.$$

$$\left.\left.\frac{2kH^2}{m^2}+\frac{T_dH}{m}\right\}\right]$$

Note: All partial derivatives have been taken after truncating the exponential terms.

REFERENCES:

[1] F.W. Harris, How many parts to make at once, Factory Manage (1913) 135-136, 152.

[2] S.K. Goyal, Economic order quantity under conditions of permissible delay in pay-ments, Journal of Operational Research Society 36 (4) (1985) 335-338.

[3] S. Chand, J. Ward, A note on economic order quantity under conditions of permis-Sible delay in payments, Journal of Operational Society 38 (1987) 83-84.

[4] K.J. Chung, A theorem on the determination of economic order quantity under Conditions of permissible delay in payments, Computer and Operations Research 25(1998) 49-52.

[5] K.J. Chung, Economic order quantity model when delay in payments is permissible,Journal of Information Optimization Science 19 (1998) 411-416.

[6] H. Hwang, S.W. Shinn, Retailer's pricing and lot sizing policy for exponentially dete-riorating products under the condition of permissible delay in payments, Computer Operation Research 24 (1997) 539-547.

[7] A.M.M. Jamal, B.R. Sarker, S. Wang, Optimal payment time for a retailer under permitted delay in payments by the wholesaler, International Journal of Production Economics 66 (2000) 59-66.
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[8] B.R. Sarker, A.M.M. Jamal, S. Wang, Optimal payment time under permissible delay in payment for products with deterioration, Production Planning and Control 11 (2001) 380-326.

[9] R.P. Covert, G.C. Philip, An EOQ model for items with weibull distribution deterioration, A.I.I.E. Transactions 5 (1973) 323-326.

[10] G.C. Philips, A generalized EOQ model for items with weibull distribution, A.I.I.E.Transactions 6 (1974) 159-162.

[11] K.S. Wu, L.Y. Ouyang, C.T. Yang, An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial back-logging, International Journal of Production Economics 101 (2006) 369-384.

[12] M.K. Meher, G.C. Panda, S.K. Sahu, An inventory model with weibull deterioration rate under the delay in payment in demand decling market, Applied Mathematical Sciences 6 (23) (2012) 1121-1133.
[13] C.K. Tripathy, U. Mishra, Ordering policy for linear deteriorating items for declining demand with permissible delay in payments, International Journal of open Problem Computational Mathematics 4 (3) (2011) 152-161.
[14] C.K. Tripathy, L.M. Pradhan, An EOQ model for three parameter weibull deteriora-tion with permissible delay in payments and associated salvage value, International Journal of Industrial Engineering and Computations 3 (2012) 115-122.
[15] R.U. Kumar, K.V. Geetha, A replenishment policy for non-instantaneous deteriorat-ing inventory system with partial backlogging, Tamsui Oxford Journal of Mathematical Sciences 2 (53) (2009) 313-332.
[16] T.J. Singh, S.R. Singh, R. Dutt, An EOQ model for perishable items with power demand and partial backlogging, International Journal of Operations and Quality Management 15 (1) (2009) 65-72.
[17] M. Basu, S. Sinha, An inationary inventory model with time dependent demandwith weibull distribution deterioration and partial backlogging under permissible delay in payments, Control and Cybernetics 36 (1) (2007) 203-217.
[18] S.K. Patra, T.K. Lenka, P.C. Ratha, An order level EOQ model for deteriorating items in a single warehouse system with price dependent demand in non-linear form,International Journal of Computational and Applied Mathematics 5 (3) (2010) 277-288.
[19] R.P. Tripathi, An inventory model with shortage and exponential demand rate under permissible delay in payments, International Journal of Management Science and Engineering Management 7 (2) (2012) 135-140.
[20] K.S. Park, Inventory models with partial backorders, International Journal of Systems Science 13 (1982) 1313-1317.
[21] R.H. Hollier, K.L. Mak, Inventory replenishment policies for deteriorating items in a decling market, International Journal of Production Economics 21 (1983) 813-826.
[22] S.K. Goyal, B.C. Giri, The production-inventory problem of a product with time varying demand, production and deterioration rate, European Journal of Operational Research 147 (2003) 549-557.
[23] K.L. Hou, An inventory model for deteriorating items with stock-dependent consumption rate and shortages under ination and time discounting, European Journal 774 Thai J. Math. Vol. 18 (2020) /R. P. Tripathi and A. K. Uniyal of Operational Research 168 (2006) 463-474.
[24] H.L. Yang, J.T. Tenj, M.S. Chern, An inventory model under ination for deteriorating items with stock-dependent consumption rate and partial backlogging shortages,International Journal of Production Economics 123 (2010) 8-19.
[25] S.S. Sana, Optimal selling price and lot size with time varying deterioration and partial backlogging, Applied Mathematics and Computation 217 (2010) 185-194.
[26] F. Ye, X. Xu, Cost allocation model for optimizing supply chain inventory with controllable lead time, Computers and Industrial Engineering 59 (2010) 93-99.
[27] M.K. Nasab, I. Konstantaras, An inventory model with stochastic review interval and special sale offer, European Journal of Operational Research 227 (2013) 81-87.
[28] K.S. Konstantaras, S. Papachristos, J.T. Teng, Supply chain models for deteriorating products with ramp type demand rate under permissible delay in payments, Expert Systems with Applications 38 (2011) 14861-14869.
[29] H.M. Wee, A deterministic lot-size inventory model for deteriorating items with shortages and declining market, Computers and Operations Research 22 (1995) 345-356.
[30] W.A. Donaldson, Inventory replenishment policy for a linear trend in demand: an analytical solution, Operations Research Quarterly 28 (1977) 663-670.
[31] H. Hariga, Optimal EOQ models for deteriorating items with time varying demand, Journal of Operations Research Society 47 (1996) 1228-1246.
[32] E.H. Silver, H.C. Meal, A simple modi_cation of the EOQ for the case of varying demand rate, Production Inventory Management 10 (1969) 52-65.

[33] K. Skouri, S. Papachristos, Four Inventory models for deteriorating item with time varying demand and partial backlogging: a cost Comparison, Optimal Control Applied Mathematics 24 (2003) 315-330.

[34] K. Skouri, I. Konstantaras, S. Papachristos, I. Ganas, Inventory methods with ramp type demand rate, partial backlogging and Weibull deterioration rate, European Journal of Operational Research 192 (2009) 79-92.

[35] S.S. Sana, Optimal selling price and lot size with time varying deterioration and partial backlogging, Applied Mathematics and Computations 217 (2010) 185-194.