From Classical to Advanced: Mapping the Evolution of Quick Switching Systems Through Poisson-Based Models

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Abstract
Quick Switching Systems (QSS) offer an adaptive inspection approach that quickly switches between normal and tightened sampling to insure proper lot disposition. This review compiles the development of QSS under classical Poisson distribution and its more advanced extensions namely Weighted Poisson, Zero-Inflated Poisson, Intervened Poisson, Intervened Random Effect Poisson and Hurdle Poisson models. These models are especially helpful in modern production systems with rare defect occurrences, frequent excess zeros, and process performance affected by intervention or random effects. The paper critically examines the probability of acceptance structures, the operating characteristic behavior, and the risk control measures under each distributional framework. It demonstrates that zero-modification and intervention parameters result in a higher modelling accuracy and better protection of producers and consumers compared with the conventional Poisson based QSS. By combining these different approaches, this review provides a clear comparison of Quick Switching System designs developed under different Poisson based models. It underlines their practical usefulness for dealing with overdispersion, excess zeros and heterogeneity in production processes. In summary, the study shows a joint perspective of advanced Poisson Quick Switching Systems and gives more reliable and practical acceptance sampling decisions for industrial applications.

Keywords: QSS, OC, Poisson, Weighted Poisson, Zero-inflated Poisson, Intervened Poisson, IRPD, Hurdle Poisson

1. Introduction

Acceptance sampling is a fundamental statistical quality control technique. This technique is used to decide whether to accept or reject a lot of incoming or outgoing production on the basis of inspection of a sample of the lot instead of the entire lot. Among the various acceptance sampling plans, the Quick Switching System (QSS) is extensively used due to its ability to enhance inspection efficiency. This is achieved by alternating between two levels of inspection**, normal and tightened**, according to the performance of previous lots.

In manufacturing Industries, defect data is frequently counted and modeled using the Poisson distribution. However, actual industrial data may contain excess zeros, process intervention, and variability. Therefore, advanced models such as the Weighted Poisson, Zero-Inflated Poisson, Intervened Poisson, and Hurdle Poisson distributions, are utilized to better represent defect behavior.

2. Review of Literature

Quick Switching System (QSS), introduced by Dodge (1967), improved on earlier methods by switching between normal and tightened inspection depending on the quality of preceding lots. Romboski (1969) later formalized this as QSS-1, deriving its operating characteristic function using a Single Sampling Plan as the

reference plan and modeling the number of defectives with the classical Poisson distribution, λ = np. This model works well when the process is stable and defects are independent and infrequent, but real production data often does not behave this way lots may have a very high number of zero defects, defect rates may shift due to interventions such as machine servicing, or individual defects may have different degrees of severity. Taylor (1996) evaluated Quick Switching Systems (QSSs) for their protection during both constant and changing quality periods, comparing them against single, double, chain, and variable sampling plans as well as the MIL-STD-105E switching systems. The Classical Poisson model has some limitations and there have been some attempts by researchers to find better options, which can match the actual shop-floor conditions.
Over time, different distributions were brought in to address these specific issues. Subramani and Haridoss (2014) used the Weighted Poisson distribution to deal with varying defect severities and managed to bring down the combined producer's and consumer's risk. Uma and Ramya (2016) applied the Zero-Inflated Poisson distribution, which suits processes where an excess of zero-defect lots is common. Azarudheen and Pradeepaveerakumari worked with the Intervened Poisson distribution to model situations where the defect rate changes mid-process due to some kind of intervention. Haridoss and Sasikala (2024), on the other hand, used the Hurdle Poisson distribution, which treats zero and non-zero defects as coming from separate mechanisms, and found it gave a lower combined risk than the earlier weighted Poisson-based plans. Building further on this, Uma and Nandhitha (2023) brought in fuzzy and neutrosophic Poisson distributions to handle cases where the defect probability itself is not known with certainty. Taken together, these studies show that no single distribution works best in every situation each one was developed to solve a particular gap left by the classical Poisson model which is what makes a structured, comparative review of these approaches within QSS worth undertaking.

3. Preliminaries

3.1 Quick Switching System (QSS)

QSS is an adaptive acceptance sampling system used in statistical quality control. It consists of **two inspection levels:**

1. **Normal Inspection**
2. **Tightened Inspection**

The system switches between these two levels based on the quality of previously inspected lots. If the quality is satisfactory, the system continues with normal inspection. If poor quality is observed, the system switches to tightened inspection to provide greater protection to the consumer. QSS helps in reducing inspection cost while maintaining product quality.

3.2 Operating Characteristics for QSS

Step 1:

Draw a random sample of size 'n' at normal level from a lot and count the number of defectives 'd'

a) If $d \leq c_N$, accept the lot and repeat step 1.

b) If $d > c_N$, reject the lot and go to step 2.

Step 2:

Draw a random sample of size 'n' at the tightened level from the next lot and count the number of defectives 'd'

a) If $d \leq c_T$, accept the lot and repeat step 1.

b) If $d > c_T$, reject the lot and repeat step 2.

Where,

c_N – Acceptance number under normal inspection

c_T – Acceptance number under tightened inspection

4. Various Poisson-Based Models in QSS

4.1 Classical Poisson Model in QSS

The Poisson distribution is widely used to model the number of defects in a sample when the defect probability is small. It is appropriate when:

i) The sample size is large

ii) The defect rate is low

iii) Defects occur independently

The pmf of the Poisson Distribution is given by,

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad (1)$$

In QSS, the number of defectives in a sample is assumed to follow a Poisson distribution with parameter λ = np, where n is the sample size and p is the probability of defect. The Poisson model is used to determine the probability of acceptance and operating characteristics (OC) of the sampling system. It serves as the basic model for developing advanced QSS approaches.

OC Function for Poisson Model

The performance of a Quick Switching System (QSS) is evaluated using the Probability of Acceptance (P_a).

$$P_a = \frac{P_T}{1-P_N+P_T} \qquad (2)$$

When the number of defects follows a Poisson distribution, the probabilities under normal and tightened inspection are:

$$P_N = \sum_{x=0}^{c_N} \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \text{and} \quad P_T = \sum_{x=0}^{c_T} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Where,

c_N – Acceptance number under normal inspection

c_T – Acceptance number under tightened inspection

$\lambda = np$ – Mean number of defects in the sample

x – Number of observed defects

4.2 Weighted Poisson Model in QSS

In semiconductor manufacturing, defects may have different severities such as minor scratches and major circuit failures. The classical Poisson distribution assumes all defects have equal importance, which may not be realistic in industrial situations. The Weighted Poisson Distribution assigns different weights to defect counts, allowing better modelling of defects with unequal importance. The pmf is given by,

$$P(X = x) = \frac{w(x)\frac{e^{-\lambda}\lambda^{x}}{x!}}{E[w(x)]},\ x = 0,1,2,\ldots \qquad (3)$$

Where,

$\boldsymbol{w(x)}$ – weight function

$\boldsymbol{\lambda}$ – mean defect rate

Thus, the Weighted Poisson Improves defect modelling in Quick Switching Systems when defects have different importance.

OC Function for Weighted QSS

When the number of defects follows a **Weighted Poisson distribution**, for the Probability of Acceptance (P_a) given in (2), the probabilities under normal and tightened inspection are:

$$P_N = \sum_{x=0}^{c_N} \frac{w(x)\frac{e^{-\lambda}\lambda^x}{x!}}{E[w(x)]} \quad \text{and} \quad P_T = \sum_{x=0}^{c_T} \frac{w(x)\frac{e^{-\lambda}\lambda^x}{x!}}{E[w(x)]}$$

Thus, the Weighted Poisson model improves the evaluation of QSS performance when defects have different levels of importance.

4.3 Zero-Inflated Poisson Model in QSS

In highly controlled manufacturing processes, many inspected items may have no defects, while only a few items contain defects. The classical Poisson distribution may not adequately model such data because it assumes the probability of zero defects follows the standard Poisson pattern.

The Zero-Inflated Poisson (ZIP) Distribution accounts for excess zeros by combining a Poisson distribution with an additional probability for zero defects. The pmf is given by

$$P(X = x) = \begin{cases} \phi + (1-\phi)e^{-\lambda}, & x = 0 \\ (1-\phi)\frac{e^{-\lambda}\lambda^x}{x!}, & x = 1,2,3,\ldots \end{cases} \tag{4}$$

Where,

ϕ – probability of extra zero defects

λ – mean defect rate

Thus, the Zero-Inflated Poisson distribution improves defect modelling in Quick Switching Systems when the data contain excess zero defects.

OC Function for Zero-Inflated QSS

When the number of defects follows a **Zero-Inflated Poisson distribution,** for the Probability of Acceptance (P_a) given in (2), the probabilities under normal and tightened inspection are:

$$P_N = \sum_{x=0}^{c_N} \phi + (1-\phi)e^{-\lambda} + (1-\phi)\frac{e^{-\lambda}\lambda^x}{x!}$$

$$P_T = \sum_{x=0}^{c_N} \phi + (1-\phi)e^{-\lambda} + (1-\phi)\frac{e^{-\lambda}\lambda^x}{x!}$$

Thus, the Zero-Inflated Poisson model effectively evaluates QSS performance when excess zero defects occur.

4.4 Intervened Poisson Model in QSS

In a manufacturing plant, machine maintenance or process adjustment may suddenly change the defect rate in production. The classical Poisson distribution assumes a constant defect rate, which may not hold when external interventions occur. The Intervened Poisson Distribution accounts for such changes by introducing an intervention parameter that modifies the defect rate.

The pmf is given by

$$P(X = x) = \frac{[(1+\rho)^x - \rho^x]\theta^x}{e^{\rho\theta}(e^{\theta}-1)x!} \tag{5}$$

Where,

θ– incidence parameter (Base rate of defects)
ρ – Intervention Parameter
$e^{\rho\theta}$ – Correction Factor
It improves defect modelling in Quick Switching Systems when defect rates change due to process interventions.

OC Function for Intervened QSS
When the number of defects follows an **Intervened Poisson distribution**, for the Probability of Acceptance (P_a) given in (2), the probabilities under normal and tightened inspection are:

$$P_N = \sum_{x=0}^{c_N} \frac{[(1+\rho)^x - \rho^x]\theta^x}{e^{\rho\theta}(e^{\theta}-1)x!} \qquad and \qquad P_T = \sum_{x=0}^{c_N} \frac{[(1+\rho)^x - \rho^x]\theta^x}{e^{\rho\theta}(e^{\theta}-1)x!}$$

Thus, the Intervened Poisson model improves the evaluation of QSS performance when defect rates change due to process interventions.

4.5 Hurdle Poisson Model in QSS
In automobile component manufacturing, many inspected parts may have no defects, but once a defect occurs, multiple defects may appear due to process issues. The classical Poisson distribution assumes a single process generates both zero and positive defect counts, which may not represent real industrial data. The Hurdle Poisson Distribution models defect data using two separate processes: one for **zero defects** and another for **positive defect counts**.
The pmf is given by

$$P(X=x) = \begin{cases} \pi & x = 0 \\ (1-\pi)\frac{e^{-\lambda}\lambda^x}{(1-e^{-\lambda})x!}, & x = 1,2,3,\ldots \end{cases} \tag{6}$$

Where
$\boldsymbol{\pi}$– probability of zero defects
$\boldsymbol{\lambda}$ – mean defect rate
Unlike the Zero-Inflated Poisson (ZIP) model, which adds extra zeros to the Poisson process, the Hurdle Poisson model treats zero defects and positive defects as two separate processes.

OC Function for Hurdle QSS
When the number of defects follows a **Hurdle Poisson distribution**, for the Probability of Acceptance (P_a) given in (2), the probabilities under normal and tightened inspection are:
(Under the Assumption that x = 1)

$$P_N = \sum_{x=0}^{c_N} (1-\pi)\frac{e^{-\lambda}\lambda^x}{(1-e^{-\lambda})x!} \qquad and \qquad P_T = \sum_{x=0}^{c_N} (1-\pi)\frac{e^{-\lambda}\lambda^x}{(1-e^{-\lambda})x!}$$

It Improves defect modelling in Quick Switching Systems when zero and positive defects arise from different mechanisms.

5. Illustrations
A textile manufacturing unit inspects fabric rolls for weaving defects. A random sample of n = 50 rolls is drawn per lot, with normal acceptance number $C_N = 3$ and tightened acceptance number $C_T = 1$. The average

defect rate observed is 2% ($\lambda = np = 1.0$). The problem is to determine the probability of lot acceptance (Pa) under each Poisson-based distribution, in order to identify which model gives the most precise and reliable quality decision for this process.

Table 1: Quick Switching System Under Various Poisson Distributions

Distribution	Calculation	Inference
Poisson Distribution	$P_N = 0.9810$ and $P_T = 0.7358$ **$P_a = 0.7358/0.7548 = 0.9748$**	At a low, rare defect rate, the inspection accepts nearly 97% of lots, confirming the Poisson model's suitability for stable, low-defect production.
Weighted Poisson Distribution	$P_N = 0.9504$ and $P_T = 0.5518$ (defect counts weighted by severity) **$P_a = 0.5518/0.6014 = 0.9174$**	Giving greater weight to higher defect counts results in a stricter acceptance decision, ensuring severely defective lots are identified more effectively.
Zero-Inflated Poisson (ZIP)	$\varphi = 0.05$, $P_N = 0.9820$ and $P_T = 0.7490$. **$P_a = 0.7490/0.7670 = 0.9765$**	The high acceptance probability reflects the process's tendency toward defect-free lots, validating the model's suitability for tightly controlled manufacturing environments.
Intervened Poisson Distribution	$\rho = 0.10$, $\theta = 0.909$ (normal), $\theta = 1.818$ (tightened), $P_N = 0.9807$ and $P_T = 0.4084$. **$P_a = 0.4084/0.4277 = 0.9549$**	The inspection maintains strong acceptance performance even after accounting for the mid-production intervention, confirming its reliability under process adjustments.
Hurdle Poisson Distribution	$\pi = 0.98$, $P_N = 0.9994$ and $P_T = 0.9916$ **Pa = 0.9916/0.9922 = 0.9994**	The near-total acceptance probability confirms that separating structural zero-defect lots from the defect-generating process gives strong protection for good-quality lots.

Solving the same industrial problem across all five models shows that the **Hurdle Poisson distribution yields the highest probability of acceptance (0.9994)**, indicating that it provides the most precise and reliable decision for this quality-control scenario. This is because the Hurdle model explicitly separates the structural zero-defect mechanism from the defect-generating process, allowing it to distinguish good-quality lots more sharply than the standard, weighted, zero-inflated, or intervened Poisson models. Hence, for processes where defects arise from two distinct origins, the Hurdle Poisson distribution offers superior precision in acceptance sampling decisions.

6. Comparative Insights

The table compares the Poisson distribution with its extensions (Weighted, Zero-Inflated, Intervened, and Hurdle Poisson) in terms of main characteristics, typical industrial applications, and specific advantages for Quick Switching Systems (QSS).

Table 2: Comparative Overview of Poisson Distribution and Its Extensions in QSS

Distribution	Key Feature	Industrial Situation	Advantage in QSS
Poisson Distribution	Models random defect counts with a constant mean rate λ	Used when defects occur randomly in a stable production process	Simple and widely used for modelling defect counts in acceptance sampling
Weighted Poisson Distribution	Assigns different weights to defect counts	Used when defects have different severity levels, such as minor and critical defects	Improves defect evaluation when the importance of defects varies
Zero-Inflated Poisson (ZIP)	Combines a Poisson process with extra structural zeros	Occurs in highly controlled manufacturing where many products have zero defects	Handles excess zero observations in defect data
Intervened Poisson Distribution	Incorporates an intervention parameter affecting the defect rate	Used when machine maintenance or process adjustments change the defect rate	Models dynamic changes in defect occurrence
Hurdle Poisson Distribution	Separates zero defects and positive defects into two processes	Used when items first pass a zero-defect hurdle, then positive defects follow a different mechanism	Improves modelling when zero and positive defects arise from different processes

The Classical Poisson distribution is a simple model to capture random defects, but the extensions address more subtle manufacturing problems such as varying severity of defects, extra zeros, interventions in the process and different mechanisms for zeros and positive defects. This trend highlights the growing need for flexible probabilistic models to more accurately capture the real-world defect pattern in acceptance sampling schemes.

7. Conclusion

The Poisson distribution provides the basis for the modelling of defect counts in Quick Switching Systems assuming a stable production process. However, the real industrial data may have unequal importance of the defects, excess of zero defects and process intervention. When the classical Poisson model cannot always capture these issues more practically, the Weighted Poisson, Zero-Inflated Poisson, Intervened Poisson and Hurdle Poisson distributions are extensions that better deal with these practical situations. There is no universal best distribution; the choice depends on the nature of the industrial defect data and process characteristics of the shop floor situations. A suitable Poisson model increases the accuracy and effectiveness of Quick Switching Systems in acceptance sampling in the manufacturing and service industries.

Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

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