

Uniqueness Results Between the Derivatives of a Linear Shift and Meromorphic Functions

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Abstract:

The uniqueness of a meromorphic function $f(z)$ and its shift $f(z + c)$ under various value-sharing and growth conditions has been extensively studied. In particular, the uniqueness of their derivatives under IM and CM value-sharing conditions has received considerable attention. In this paper, we investigate the uniqueness of $f^{(l)}(z)$ and $L^{(k)}(f)$ when they share two distinct values CM. Where $L(f) = \sum_{j=1}^n a_j f(z + c_j)$ be a linear shift function of $f(z)$. Our results refine and extend existing uniqueness theorems. We also study the role of deficiency conditions in value sharing and their impact on uniqueness. Finally, we establish uniqueness results for $f^{(l)}(z)$ and $L^{(k)}(f)$ sharing one value CM under suitable deficiency conditions, and obtain the corresponding IM result under a stronger deficiency condition.

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1. Introduction

Throughout this paper, we assume that the reader is familiar with the standard notation of Nevanlinna theory, as presented, for example, in [1–3]. In particular, $T(r, f)$, $m(r, f)$, and $N(r, f)$ denote the characteristic, proximity, and counting functions, respectively. We denote by $S(r, f)$ the class of all small functions of f ; that is, a meromorphic function a belongs to $S(r, f)$ if $T(r, a) = o(T(r, f)), r \rightarrow \infty$. Nevanlinna theory has found extensive applications in the study of meromorphic solutions of complex differential equations. Subsequently, a difference analogue of the logarithmic derivative was developed, leading to the emergence of difference equations in complex analysis. For further details and related results, we refer the reader to [4,5].

Definitions [19]. Let F and G be two non-constant meromorphic functions such that F and G share the value 1 IM. Let z_1 be a 1-point of F of order m , a 1-point of G of order n . We denote by $\bar{N}_L\left(r, \frac{1}{F-1}\right)$, the counting function of those 1-points of F where $m > n$; by $N_E^{(1)}\left(r, \frac{1}{F-1}\right)$ the counting function of those 1-points of F where $m = n = 1$; by $N_E^{(2)}\left(r, \frac{1}{F-1}\right)$ the counting function of those 1-points of F where $m = n \geq 2$; in each of the function, each 1-point is counted only once. We define $\bar{N}_L\left(r, \frac{1}{G-1}\right)$, $N_E^{(1)}\left(r, \frac{1}{G-1}\right)$ and $N_E^{(2)}\left(r, \frac{1}{G-1}\right)$

in the same way. In addition, $\bar{N}_{(2)}\left(r, \frac{1}{F}\right)$ is the counting function of zeros of F whose multiplicities are greater than 2. By the above definitions, we have

$$\bar{N}\left(1, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{F}\right) \leq N\left(r, \frac{1}{F}\right). \quad (1.1)$$

Also, define $\delta(0, F) = 1 - \overline{\lim}_{n \rightarrow \infty} \frac{N\left(r, \frac{1}{F}\right)}{T\left(r, \frac{1}{F}\right)}$, $\Theta(\infty, F) = 1 - \overline{\lim}_{n \rightarrow \infty} \frac{\bar{N}(r, F)}{T(r, F)}$.

Theorem 1 ([12]). Let $f(z)$ be a finite order transcendental entire function, and let $a (\neq 0) \in \mathbb{C}$. If $f'(z)$ and $f(z + c)$ share 0 CM and a IM, then $f'(z) = f(z + c)$.

In 2021, Z. Wang, X. Qi ([13]) improved this result for the entire function, and they proved the following:

Theorem 2 ([13]). Let $f(z)$ be a transcendental entire function of hyper-order strictly less than 1, and let $a \neq 0 \in \mathbb{C}$. If $f'(z)$ and $f(z + c)$ share a IM and $\delta(0, f) > \frac{4}{5}$. Then $f'(z) = f(z + c)$.

In this view, W.-J. Chen, Z.-G. Haunag [14] worked on the unicity of j th derivative of a meromorphic function $f(z)$ and k th derivative of its shift $f(z + c)$ for partially shared value and they proved the following result:

Theorem 3 ([14]). Let $f(z)$ be a non-constant meromorphic function of finite order, and let c be a non-zero finite complex number and j, k be integers with $0 \leq j < k$. If $f^{(j)}(z)$ and $f^{(k)}(z + c)$ share a non-zero finite complex number a CM, and satisfy $E(0, f^{(j)}(z)) \subset E(0, f^{(k)}(z + c))$ and $E(\infty, f^{(k)}(z + c)) \subset E(\infty, f^{(j)}(z))$, then $f^{(j)}(z) = f^{(k)}(z + c)$.

In the year 2025, Pramanik, Dey discussed the deficiency condition on Theorem 3 to get the following results:

Theorem 4 ([19]). Let $f(z)$ be a non-constant meromorphic function of hyper-order less than 1 and let c be a non-zero complex number, and let j, k be integers with $0 \leq j < k$. Suppose that $f^{(j)}(z)$ and $f^{(k)}(z + c)$ share $0, \infty$ CM and satisfy

$$\overline{\lim}_{n \rightarrow \infty} \frac{N\left(r, \frac{1}{f^{(j)}-1}\right)}{T(r, f^{(j)})} < \lambda < 1, \text{ then } f^{(j)}(z) = f^{(k)}(z + c).$$

Theorem 5 ([19]). Let f be a transcendental meromorphic function of hyper-order less than 1, let $c \in \mathbb{C} \setminus \{0\}$, and let j, k be integers with $0 \leq j < k$. Suppose that $f^{(j)}(z)$ and $f^{(k)}(z + c)$ share $a \neq 0, \infty$ IM and satisfy

$$5\delta(0, f) + (7 + 2j + 2k)\Theta(\infty, f) > 11 + 2j + 2k,$$

then $f^{(j)}(z) = f^{(k)}(z + c)$.

Theorem 6. Let $f(z)$ be a non-constant meromorphic function of hyper-order less than 1 and let $L(f) = \sum_{j=1}^n a_j f(z + c_j)$ be a linear shift function of $f(z)$, where $a_j \neq 0, j = 1, 2, \dots, n, c_j \in \mathbb{C} (j = 1, 2, \dots, n)$ are distinct complex numbers and let l, k be integers with $0 \leq l < k$. Suppose that $f^{(l)}(z)$ and $L^{(k)}(f)$ share $0, \infty$ CM and satisfy

$$\overline{\lim}_{n \rightarrow \infty} \frac{N\left(r, \frac{1}{f^{(l)}-1}\right)}{T(r, f^{(l)})} < \lambda < 1, \text{ then } f^{(l)}(z) = L^{(k)}(f).$$

Theorem 7. Let f be a transcendental meromorphic function of hyper-order less than 1, let $L(f)$ is as defined in Theorem 6 and let l, k be integers with $0 \leq l < k$. Suppose that $f^{(l)}(z)$ and $L^{(k)}(f)$ share $a \neq 0, \infty$ IM and satisfy

$$5\delta(0, f) + (2 + k)\Theta(0, f) + (2l + k)\delta(\infty, f) > 2(l + k) + 6,$$

then $f^{(l)}(z) = L^{(k)}(f)$.

2.Lemmas

Lemma 1 ([15]). Let $f(z)$ be a non-constant meromorphic function of hyper-order $\rho_2(f) < 1$, and $c \in \mathbb{C} \setminus \{0\}$. Then

$$m\left(r, \frac{f(z+c)}{f(z)}\right) + m\left(r, \frac{f(z)}{f(z+c)}\right) = S(r, f).$$

Lemma 2([15]). Let $T: [0, +\infty) \rightarrow [0, +\infty)$ be a non-decreasing continuous function and $s \in (0, \infty)$. If hyper-order of T is strictly less than one, i.e.

$$\overline{\lim}_{n \rightarrow \infty} \frac{\log \log T(r)}{\log r} = \rho_2 < 1,$$

And $\delta \in (0, 1 - \rho_2)$, then $T(r+s) = T(r) + o(T(r)r^{-\delta})$ as $r \rightarrow +\infty$ outside some set of finite logarithmic measure.

Lemma 3 ([16]). Suppose that $f(z)$ be a non-constant meromorphic function of finite order and a_s for $s \in \{1, 2, \dots, q\}$ ($q \geq 2$) are distinct finite complex numbers. Let j, k be integers with $0 \leq j < k$. Then for each $r > 0$ we have

$$m\left(r, f^{(j)}(z)\right) + \sum_{s=1}^q m\left(r, \frac{1}{f^{(j)}(z) - a_s}\right) \leq 2T\left(r, f^{(j)}(z)\right) - N_{\text{pair}}(r, f) + S(r, f),$$

where,

$$N_{\text{pair}} = 2N\left(r, f^{(j)}(z)\right) - N\left(r, f^{(k)}(z+c)\right) + N\left(r, \frac{1}{f^{(k)}(z+c)}\right) + S(r, f).$$

Lemma 4 ([3]). Let $f(z)$ be a non-constant meromorphic function in a complex plane and k be a positive integer. Set

$$\psi(z) = \sum_{k=0}^n a_k(z) f^{(k)}(z),$$

where, $a_k(z)$ ($k \in \{0, 1, \dots, n\}$) are small functions of $f(z)$. Then

$$T(r, \psi) \leq T(r, f) + k\bar{N}(r, f) + S(r, f) \leq (k+1)T(r, f) + S(r, f),$$

$$N\left(r, \frac{1}{\psi}\right) \leq N\left(r, \frac{1}{f}\right) + k\bar{N}(r, f) + S(r, f).$$

Lemma 5 ([17]). Let f be a non-constant meromorphic function, then

$$N\left(r, \frac{1}{f^{(n)}}\right) \leq T(r, f^{(n)}) - T(r, f) + N\left(r, \frac{1}{f}\right) + S(r, f),$$

$$N\left(r, \frac{1}{f^{(n)}}\right) \leq N\left(r, \frac{1}{f}\right) + n\bar{N}(r, f) + S(r, f).$$

Lemma 6 ([18]). Let

$$H(z) = \left(\frac{F''}{F'} - \frac{2F'}{F-1}\right) - \left(\frac{G''}{G'} - \frac{2G'}{G-1}\right), \quad (2.1)$$

where F and G be two non-constant meromorphic functions. If F and G share 1 IM and $H \neq 0$, then

$$N_E^{(1)}\left(r, \frac{1}{F-1}\right) \leq N(r, H) + S(r, G) + S(r, F). \quad (2.2)$$

3. Main Results

Proof of Theorem 6. Suppose that $f^{(l)}(z) \neq L^{(k)}(f)$. Since $f^{(l)}(z)$ and $L^{(k)}(f)$ share $0, \infty$ CM. By the second fundamental theorem we have,

$$\begin{aligned} T(r, f^{(l)}) &\leq N(r, f^{(l)}) + N\left(r, \frac{1}{f^{(l)}}\right) + N\left(r, \frac{1}{f^{(l)} - 1}\right) + S(r, f^{(l)}) \\ &\leq N(r, L^{(k)}) + N\left(r, \frac{1}{L^{(k)}}\right) + N\left(r, \frac{1}{L^{(k)} - 1}\right) + S(r, L^{(k)}) \\ &\leq 2T(r, L^{(k)}) + \lambda T(r, f^{(l)}) + S(r, f^{(l)}), \end{aligned}$$

where $\lambda < 1$. This implies

$$T(r, f^{(l)}) = O(T(r, L^{(k)})) \quad (3.1)$$

By Lemma 2

$$T(r, L^{(k)}) = T(r, f^{(l)}) + O(1),$$

By Lemma 4

$$T(r, L^{(k)}) \leq (k+1)T(r, f) + S(r, f). \quad (3.2)$$

From (3.1) and (3.2),

$$S(r, f^{(l)}) = S(r, L^{(k)}) = S(r, f).$$

Using Lemma 3

$$\begin{aligned} T(r, f^{(l)}) &\leq N(r, f^{(l)}) + N\left(r, \frac{1}{f^{(l)}}\right) + N\left(r, \frac{1}{f^{(l)} - 1}\right) - N_{\text{pair}}(r, f) + S(r, f) \\ &= N\left(r, \frac{1}{f^{(l)}}\right) + N\left(r, \frac{1}{f^{(l)} - 1}\right) - N(r, f^{(l)}) + N(r, L^{(k)}) - N\left(r, \frac{1}{L^{(k)}}\right) + S(r, f) \\ &\leq N\left(r, \frac{1}{f^{(l)} - 1}\right) + S(r, f) \end{aligned}$$

Hence $T(r, f^{(l)}) \leq \lambda T(r, f^{(l)}) + S(r, f)$, this yields a contradiction for $\lambda < 1$.

Proof of Theorem 7. Let

$$F(z) = \frac{f^{(l)}(z)}{a} \quad \text{and} \quad G(z) = \frac{L^{(k)}(z)}{a}. \quad (3.3)$$

Then it can be easily shown that $S(r, F) = S(r, f)$ and $S(r, G) = S(r, f)$.

Now, If $H(z) \neq 0$, then,

$$\begin{aligned} N(r, H) &\leq \bar{N}_{(2)}\left(r, \frac{1}{F}\right) + \bar{N}_{(2)}\left(r, \frac{1}{G}\right) + \bar{N}_L\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{G-1}\right) + \bar{N}(r, F) + \\ &\bar{N}(r, G) + N_0\left(r, \frac{1}{F'}\right) + N_0\left(r, \frac{1}{G'}\right) \end{aligned} \quad (3.4)$$

where $N_0\left(r, \frac{1}{F'}\right)$ and $N_0\left(r, \frac{1}{G'}\right)$ are the counting functions corresponding to the zeros of F and G that are not zeros of F and $F-1$ and G and $G-1$ respectively. But, F and G share 1 IM, it is easy to observe that

$$\begin{aligned} \bar{N}\left(r, \frac{1}{F-1}\right) &= N_E^{(1)}\left(r, \frac{1}{F-1}\right) + N_E^{(2)}\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{F-1}\right) \\ &+ \bar{N}_L\left(r, \frac{1}{G-1}\right) = \bar{N}\left(r, \frac{1}{G-1}\right) \end{aligned} \quad (3.5)$$

Hence, from (3.5) and (1.1) we have

$$\begin{aligned} \bar{N}\left(r, \frac{1}{F-1}\right) + \bar{N}\left(r, \frac{1}{G-1}\right) &= N_E^{(1)}\left(r, \frac{1}{F-1}\right) \bar{N}_L\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{G-1}\right) \\ &+ N_E^{(2)}\left(r, \frac{1}{G-1}\right) \bar{N}\left(r, \frac{1}{G-1}\right) \leq N_E^{(1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{F-1}\right) + N\left(r, \frac{1}{G-1}\right) \\ &\leq N_E^{(1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{F-1}\right) + T(r, G) + O(1). \end{aligned} \quad (3.6)$$

Using Nevanlinna second fundamental theorem, we get

$$T(r, F) + T(r, G) \leq \bar{N}\left(r, \frac{1}{F}\right) + \bar{N}(r, F) + \bar{N}\left(r, \frac{1}{F-1}\right) - N_0\left(r, \frac{1}{F'}\right) \\ + \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}(r, G) + \bar{N}\left(r, \frac{1}{G-1}\right) - N_0\left(r, \frac{1}{G'}\right) + S(r, f). \quad (3.7)$$

From (3.6) - (3.7)

$$T(r, F) \leq \bar{N}\left(r, \frac{1}{F}\right) + \bar{N}(r, F) + \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}(r, G) - N_0\left(r, \frac{1}{F'}\right) - N_0\left(r, \frac{1}{G'}\right) \\ + N_E^{(1)}\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{F-1}\right) + S(r, f) \quad (3.8)$$

Equation (3.8) along with (1.1), (3.4) and Lemma 6 gives

$$T(r, F) \leq N\left(r, \frac{1}{F}\right) + N\left(r, \frac{1}{G}\right) + 2[\bar{N}(r, F) + \bar{N}(r, G)] + \\ 2\bar{N}_L\left(r, \frac{1}{F-1}\right) + \bar{N}_L\left(r, \frac{1}{G-1}\right) + S(r, f). \quad (3.9)$$

Using Lemma 5, we have

$$\bar{N}_L\left(r, \frac{1}{F-1}\right) \leq N\left(r, \frac{1}{f}\right) + lN(r, f) + S(r, f). \quad (3.10)$$

Similarly,

$$\bar{N}_L\left(r, \frac{1}{G-1}\right) \leq N\left(r, \frac{1}{f}\right) + kN(r, f) + S(r, f). \quad (3.11)$$

Also,

$$N\left(r, \frac{1}{F}\right) \leq N\left(r, \frac{1}{f}\right) + l\bar{N}(r, f) + S(r, f), \quad (3.12)$$

and

$$N\left(r, \frac{1}{G}\right) \leq N\left(r, \frac{1}{f}\right) + k\bar{N}(r, f) + S(r, f), \quad (3.13)$$

$$\bar{N}(r, F) \leq \bar{N}(r, f) + O(1), \quad (3.14)$$

and

$$\bar{N}(r, G) \leq \bar{N}(r, f) + O(1). \quad (3.15)$$

From Lemma 5 and (3.9) - (3.15),

$$T(r, f) \leq T(r, f^{(l)}) - N\left(r, \frac{1}{f^{(l)}}\right) + N\left(r, \frac{1}{f}\right) + S(r, f) \\ \leq N\left(r, \frac{1}{G}\right) + N\left(r, \frac{1}{f}\right) + 2\left[\bar{N}(r, F) + \bar{N}(r, G) + \bar{N}_L\left(r, \frac{1}{F-1}\right)\right] + \\ \bar{N}_L\left(r, \frac{1}{G-1}\right) + S(r, f) \\ \leq 5N\left(r, \frac{1}{f}\right) + (2+k)\bar{N}\left(r, \frac{1}{f}\right) + (2l+k)N(r, f) + S(r, f), \\ \leq (5(1-\delta(0, f))T(r, f) + ((2+k)(1-\Theta(0, f))T(r, f) + \\ ((2l+k)(1-\delta(\infty, f))T(r, f) + S(r, f). \quad (3.16)$$

This implies

$$(R - (2(k+l) + 6))T(r, f) \leq S(r, f).$$

Where,

$$R = 5\delta(0, f) + (2+k)\Theta(0, f) + (2l+k)\delta(\infty, f) - (2(k+l) + 6) > 0.$$

which is a contradiction. Hence $H(z) = 0$.

From (2.1) we get,

$$\frac{1}{G-1} = \frac{c_1}{F-1} + c_2, \quad (3.17)$$

where c_1 and c_2 are integrating constants. Equation (3.17) can be written as

$$F = \frac{(c_1 - c_2)(G-1) + 1}{1 - c_2(G-1)}. \quad (3.18)$$

We get the following two cases.

Case1: Let $c_2 \neq 0, -1$. Then from (3.18)

$$\bar{N}\left(r, \frac{1}{\frac{c_2+1}{c_2}-G}\right) \leq \bar{N}(r, F).$$

Hence, by second fundamental theorem,

$$\begin{aligned} T(r, G) &\leq \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}\left(r, \frac{1}{\frac{c_2+1}{c_2}-G}\right) + \bar{N}(r, G) + S(r, f) \\ &\leq \bar{N}\left(r, \frac{1}{G}\right) + \bar{N}(r, G) + \bar{N}(r, F) + S(r, f). \end{aligned} \quad (3.19)$$

From Lemma 2, we have

$$T\left(r, L^{(k)}(f)\right) = T(r, f^{(k)}) + O(1).$$

From this, Lemma 5 and (3.19)

$$T(r, f) \leq N\left(r, \frac{1}{f}\right) + 2\bar{N}(r, f) + S(r, f), \text{ which is a contradiction.}$$

Case 2: Suppose that $c_2 = 0$. From (3.18) we have

$$G = \frac{F-(1-c_1)}{c_1}, \quad F = c_1 G - (c_1 - 1). \quad (3.20)$$

If $c_1 \neq 1$, from (3.20) we obtain

$$N\left(r, \frac{1}{G-\frac{c_1-1}{c_1}}\right) = N\left(r, \frac{1}{G}\right).$$

As in case 1, we get a contradiction. Thus $c_1 = 1$. Hence $f^{(l)}(z) = L^{(k)}(f)$.

Case 3: Suppose that $c_2 = -1$. From (3.18) we have

$$G = \frac{c_1}{(c_1+1)-F}, \quad F = \frac{-c_1+G(c_1+1)}{G}. \quad (3.21)$$

If $c_1 \neq -1$, we obtain from (3.21) that $N\left(r, \frac{1}{G-\frac{c_1}{c_1+1}}\right) = N\left(r, \frac{1}{F}\right)$. Proceeding as in Case 2,

we come to a contradiction. Hence, $c_1 = -1$. Thus from (3.21), we get $F \cdot G = 1$, that is

$$f^{(l)}(z)L^{(k)}(f) = a^2. \quad (3.22)$$

Which implies,

$$\frac{1}{(f^{(l)})^2} = \frac{L^{(k)}}{f^{(l)}} \cdot \frac{1}{a^2}. \quad (3.23)$$

Now, using the lemma of logarithmic derivative, Lemma1 and (3.23), we get

$$m\left(r, \frac{1}{f^{(l)}}\right) = \frac{1}{2}m\left(r, \frac{1}{(f^{(l)})^2}\right) = \frac{1}{2}m\left(r, \frac{L^{(k)}}{f^{(l)}}\right) = S(r, f).$$

Also

$$N\left(r, \frac{1}{f^{(l)}}\right) \leq m\left(r, \frac{L^{(k)}}{f^{(l)}}\right) = S(r, f).$$

Thus $T\left(r, \frac{1}{f^{(l)}}\right) = S(r, f)$ i.e. $T(r, f^{(l)}) = S(r, f) = S(r, f^{(l)})$,

Which is a contradiction.

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