

Relativistic Time Dilation and its Effect on the GPS System

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Abstract

A thorough analysis of relativistic time dilation and its crucial function in the precise functioning of the Global Positioning System (GPS) is presented in this work. The study looks at both gravitational time dilation (GTD) predicted by General Relativity and kinematic time dilation (KTD) resulting from Special Relativity. The application of relativistic adjustments to GPS satellite clocks is given special attention. GPS satellite clocks have a kinematic time loss of about $7.2 \mu\text{s}$ per day due to their high orbital velocity, and a gravitational time gain of about $45.7 \mu\text{s}$ per day due to their location in a weaker gravitational field. In comparison to clocks on Earth, the total effect yields a net clock gain of roughly $38.6 \mu\text{s}$ every day. The temporal error that would come from ignoring these relativistic effects would cause positioning errors of about 11 km per day, proving that relativity is a practical necessity for satellite navigation rather than just a theoretical idea.

Keywords: Gravitational time dilation (GTD), Kinematic time dilation (KTD), optical lattice clocks, GPS, Relativistic frequency correction, Sagnac correction

1. Introduction

The measurement of time is fundamental to modern navigation. According to Einstein's theory of relativity, time is not absolute and can change based on gravity and speed. This phenomenon is known as time dilation [1-4]. For example, $\Delta r = c\Delta t$. Thus, a timing error of only 1 ns corresponds to approximately $\Delta r = (2.9979 \times 10^8) \times (10^{-9} \text{ s}) \approx 0.30 \text{ m}$. Therefore, nanosecond-level clock accuracy is essential for high-precision satellite navigation. Ashby's comprehensive review [9] of relativity in GPS emphasizes that timing errors of approximately 1 ns correspond to position errors of roughly 30 cm.

GPS satellites contain atomic clocks of extremely high stability. However, an atomic clock aboard an orbiting satellite does not run at exactly the same rate as an identical clock on Earth's surface. Two principal relativistic effects are responsible:

1. Special-relativistic time dilation due to the velocity of the satellite.
2. General-relativistic gravitational time dilation due to the difference in gravitational potential between the satellite and the Earth's surface.

These effects act in opposite directions. The satellite's high orbital velocity makes its clock run slower, while its position in the weaker gravitational field makes its clock run faster. The net effect is approximately $38 \mu\text{s}$ per day with the gravitational contribution dominating. NIST gives approximately $-7 \mu\text{s/day}$ from special relativity and $+45 \mu\text{s}$ per day from general relativity, resulting in a net $+38 \mu\text{s/day}$ [5].

The purpose of this paper is to explain these effects quantitatively and demonstrate why relativity is essential to the operation of GPS.

2. Theoretical Background

Proper time in Physics is defined as the time interval measured by a clock moving along the worldline of the object that travels between two events through space and time as the observer or event.

For an object moving with velocity v in special relativity,

$$d\tau = dt \sqrt{1 - \frac{v^2}{c^2}} = \frac{dt}{\gamma} \quad (1)$$

Where, $d\tau$ is the proper time measured by the clock, dt is the coordinate time, v is the relative velocity, c is the speed of light, and γ is the Lorentz parameter.

Consequently, $d\tau < dt$ (2)

A moving clock therefore *runs more slowly* relative to an inertial reference clock.

For $v \ll c$, the Taylor series approximation gives:

$$\sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2} \quad (3)$$

and therefore $\frac{d\tau}{dt} \approx 1 - \frac{v^2}{2c^2}$ (4)

This approximation is particularly useful for calculating the special-relativistic correction to GPS satellite clocks.

3. Special-Relativistic Time Dilation (STD) of GPS Satellites

GPS satellites orbit the Earth at approximately 20,200 km above the Earth's surface. Their orbital velocity is approximately $v \approx 3.87 \text{ km.s}^{-1}$ the special-relativistic fractional clock-rate difference is approximately $\frac{\Delta\tau}{\Delta t} \approx -\frac{v^2}{2c^2}$ from equation (4). Using $v = 3.87 \times 10^3 \text{ m.s}^{-1}$ and $c = 2.998 \times 10^8 \text{ m.s}^{-1}$, we obtain $\frac{v^2}{2c^2} \approx 8.3 \times 10^{-11}$. For a time period of one single day, $\Delta t = 86400 \text{ s}$, so the total daily time shift ($\Delta\tau$) is $\Delta\tau \approx -8.3 \times 10^{-11} \times 86,400 \text{ s}$ or $\Delta\tau_{SR} \approx -7.2 \mu\text{s/day}$. Thus, because of their orbital velocity, GPS satellite clocks lose approximately 7 μs per day relative to clocks on Earth. This is the special-relativistic time-dilation effect.

4. General-Relativistic Gravitational Time Dilation (GTD)

General Relativity published by Albert Einstein in 1915, redefined gravity not as a force but as the curvature of space time caused by mass and energy [2]. One of its most striking predictions is GTD: clocks situated deeper in a gravitational potential well tick more slowly than identical clocks located at a higher altitude (where gravity is weaker). This effect is a direct consequence of the equivalence principle and has been confirmed experimentally with extraordinary precision — from the Pound–Rebka experiment (1960) to modern optical lattice clocks measuring height differences of just one millimeter. GTD is a direct consequence of the curved space time predicted by General Relativity. The equivalence-principle [6-7] argument provides an intuitive local explanation: light propagating upward in a gravitational field undergoes a redshift, indicating that clocks at higher gravitational potential run faster than clocks at lower potential. In the weak-field regime, the effect is described by $\Delta f/f \approx -\Delta\Phi/c^2$ or $\Delta f/f \approx -gh/c^2$ near Earth's surface. For a static, spherically symmetric, non-rotating gravitating body, the Schwarzschild metric gives the exact relation $d\tau/dt = \sqrt{1 - 2GM/(rc^2)}$ for a stationary observer. Thus, gravitational time dilation is not an alteration of a particular clock mechanism but a geometrical property of spacetime that affects all local

physical processes. Its experimental confirmation, from the Pound–Rebka experiment to modern optical clocks, demonstrates the accuracy of General Relativity over both macroscopic and extremely small height scales.

5. Experimental tests on evidence of Time dilation

Extensive experimental evidence [7-10] supports every component of the relativistic framework: Muon storage ring experiments (1977) verified time dilation for particles and this yields a relativistic Lorentz factor $\gamma \approx 29.3$. The Pound–Rebka experiment (1960) first measured gravitational redshift on Earth. The Hafele–Keating experiment (1971) confirmed both kinematic and gravitational time dilation with atomic clocks flown around the world. Modern optical lattice clocks have measured gravitational time dilation over 1 mm height differences (JILA, 2022), providing direct laboratory confirmation of general relativity at the millimeter scale. The ACES mission (2025–ongoing) is testing gravitational redshift with tenfold improved precision using space borne cold-caesium clock (PHARAO) and a hydrogen maser (SHM) on the ISS[12].

5.1. Muon time dilation experiments:

This provides critical empirical proof for special relativity by demonstrating that moving particles live longer than their resting counterparts. While classical physics predicts that high-altitude atmospheric muons should decay before reaching the ground, special relativistic time dilation (and length contraction) explains their survival, a phenomenon confirmed with high precision by both cosmic-ray observations and particle accelerator storage rings. The key experimental validations are given in the table below.

Experiment	Type	Velocity / (γ)	Expected vs. Observed Survival	Significance
Rossi & Hall (1941)	Cosmic-Ray (Mountain vs. Base)	($v \approx 0.994c$)	Expected flux ratio: ~ 22 Observed flux ratio: ~ 1.4	First qualitative-to-quantitative confirmation of time dilation.
Frisch & Smith (1963)	Cosmic-Ray (Mt. Washington)	($v \approx 0.995c$)	Expected: 27/hour at sea level Observed: 412/hour at sea level	Demonstrated an effective dilation factor of ~ 8.8 , matching relativity.
CERN Storage Ring (1970)	Controlled Accelerator	($v \approx 0.9994c$) ($\gamma \approx 29.33$)	Rest life: $\sim 2.197 \mu\text{s}$ Dilated life: $\sim 64.4 \mu\text{s}$	Achieved 0.2% precision in a circular path and confirmed CPT symmetry.

5.2. The Mössbauer effect

Rudolf Mössbauer discovered it in 1958 while studying the 129 keV transition in iridium-191 (^{191}Ir), for which he received the Nobel Prize in Physics in 1961. This experiment describes the recoil-free emission and absorption of gamma rays by atomic nuclei fixed in a solid crystal lattice making a key tool for testing a relativistic time dilation and gravitational red shift. The Mössbauer effect was essential because the predicted gravitational redshift for ~ 22.5 m height on Earth is only $\sim 2.5 \times 10^{-15}$ (fractional frequency shift $\Delta f/f \approx gh/c^2$). Without recoil-free emission/absorption, the gamma lines would be too broad and shifted to detect such a minuscule change.

5.3. The Pound–Rebka experiment (1959)

In general relativity, a photon traveling upward against gravity experiences a gravitational redshift (its frequency decreases, wavelength increases) because it loses energy. A photon falling downward experiences a blue shift. The predicted fractional frequency shift over a height difference (h) near Earth's surface is approximately:

$$\frac{\Delta f}{f} \approx \frac{gh}{c^2} \quad (5)$$

Where ‘ g ’ is gravitational acceleration, c is the speed of light. For typical lab heights (tens of meters), this effect is tiny—on the order of 10^{-15} —so it requires extremely high precision. Robert Pound and Glen Rebka at Harvard used the Mössbauer effect with 14.4 keV gamma rays from iron-57 (^{57}Fe) [8]. This effect allows recoil-free emission and absorption of gamma rays with very narrow line-widths, making such a small shift detectable. They placed the gamma-ray source and absorber in the Jefferson Physical Laboratory tower, separated by about 22.5–74 feet (roughly 7–22.6 meters), and measured the shift for gamma rays traveling both upward and downward. To isolate the gravitational effect, they reversed the setup and accounted for other factors like temperature.

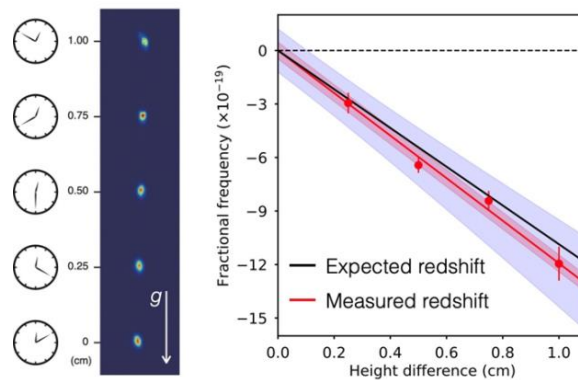


Figure.1. Gravitational time dilation measured over a sub-centimeter scale using an optical lattice atomic clock.

5.4. Hafele–Keating experiment (1971)

The Hafele–Keating experiment (1971) was a landmark test of Einstein's theories of special and general relativity using macroscopic atomic clocks [11]. Physicists Joseph C. Hafele and Richard E. Keating from the U.S. Naval Observatory flew four cesium-beam atomic clocks aboard commercial airliners on two around-the-world trips: one eastward and one westward. They then compared the flying clocks to stationary reference clocks at the U.S. Naval Observatory in Washington, D.C. The predicted and observed Relativistic Time Dilation in the Eastward and Westward time shifts are given in tabular form.

Hafele-Keating statistics	Predicted nanoseconds difference			Actual Nano-seconds difference	Difference
	Gravitational (General Relativity)	Kinematic (Special Relativity)	Total		
Eastward	+144 ± 14	-184 ± 18	-40 ± 23	-59 ± 10	0.76
Westward	+179 ± 18	+96 ± 10	+275 ± 21	+273 ± 7	0.09

6. Net Relativistic Effect on GPS Clocks

The two principal relativistic effects are: Approximate clock-rate change with Special relativity: due to change in orbital velocity— $7 \mu\text{s/day}$, and General relativity: due to change in gravitational potential

+45 $\mu\text{s/day}$ Net effect is +38 $\mu\text{s/day}$. Therefore, the atomic clocks aboard GPS satellites tend to run approximately 38 microseconds faster per day than corresponding clocks on Earth if no relativistic correction is applied.

7. GPS Position Determination

A GPS receiver determines its position by receiving signals from multiple satellites. For a given satellite, the basic propagation relation can be written approximately as

$$c(t_i' - t_i) = |\mathbf{r} - \mathbf{r}_i| + c\Delta t, \quad i = 1, 2, 3, 4 \dots N \quad (6)$$

where $\mathbf{r}_i$ is the position vector of the satellite, t_i is the signal transmission time according to the satellite clock, $\mathbf{r}$ is the receiver position, t_i' is the signal reception time. Following Ashby's comprehensive treatment of relativity in GPS [9], a minimum of four simultaneous equations ($N \geq 4$) are required to uniquely resolve the receiver's three spatial coordinates and its clock offset. Consequently, the accuracy of GPS positioning depends directly on accurate synchronization of satellite and receiver clocks.

8. Relativistic Frequency Correction

Instead of allowing the satellite clock to accumulate the 38 $\mu\text{s/day}$ difference, GPS incorporates relativistic corrections into the satellite clock system. The fractional frequency correction associated with the main relativistic effects is approximately

$$\frac{\Delta f}{f} \approx -\frac{v^2}{2c^2} + \frac{GM}{c^2 \left(\frac{1}{R_E} - \frac{1}{r_s} \right)} \quad (7)$$

where R_E represents the mean radius of the Earth, and r_s represents the orbital radius of the satellite. The first term represents special-relativistic time dilation and the second represents gravitational time dilation. The resulting fractional correction is approximately

$$\frac{\Delta f}{f} \approx -4.465 \times 10^{-10} \quad (8)$$

The correction is of such importance that the nominal frequency of the satellite clocks is offset before launch so that the clock produces the desired GPS system time while in orbit.

9. Effect of Orbital Eccentricity

The simplified calculation above assumes a circular orbit. Actual satellite orbits are not perfectly circular. Because the satellite's velocity and gravitational potential vary during an elliptical orbit, the relativistic clock correction also varies periodically. A commonly used relativistic correction associated with orbital eccentricity has the vector form $\Delta t_r = -\frac{2}{c^2}(\mathbf{r} \cdot \mathbf{v})$. This equation calculates the periodic relativistic clock correction for GPS in eccentric (non-circular) orbits which is driven by $\mathbf{r} \cdot \mathbf{v}$ when it is converted in orbital form, the equation becomes

$$\Delta t_r = -\frac{2\sqrt{\mu a}}{c^2} e \sin E \quad (9)$$

where $\mu = GM$, a is the semi-major axis, e is orbital eccentricity, E is the eccentric anomaly. This correction is important for precise GPS positioning and timing. Ashby's review discusses eccentricity-related relativistic effects and the detailed treatment required for GPS satellite clocks.

10. The Sagnac Effect

The Sagnac effect is a relativistic phenomenon arising from the rotation of the reference frame. In GPS, the Earth rotates while the signal travels from the satellite to the receiver. During the signal transit time (approximately 65–85 ms), the receiver moved eastward by up to about 30 meters. This changes the actual path length, compared to a non-rotating Earth, introducing an additional time correction [9]. For a signal

travelling from satellite S to receiver R on the Earth's surface, the Sagnac correction Δt_{Sag} (in seconds) is given by:

$$\Delta t_{Sag} = \frac{2\omega_E}{c^2} A \quad (10)$$

where $\omega_E = 7.292115 \times 10^{-5} \text{ rad/s}$ is the Earth's angular velocity, and A is the area projected onto the equatorial plane by the vectors from the Earth's axis to the satellite and to the receiver [7]. In Cartesian earth-centered coordinates (with the z-axis along the rotation axis), the correction can be written as:

$$\Delta t_{Sag} = \frac{2\omega_E}{c^2} (x_R y_S - y_R x_S) \quad (11)$$

where (x_R, y_R) and (x_S, y_S) are the equatorial coordinates of receiver and satellite, respectively. The Sagnac correction typically ranges from 30 to 120 nanoseconds, depending on the satellite–receiver geometry. An uncorrected Sagnac effect would produce a position error of roughly 10–30 meters – unacceptable for modern GPS applications [9]. The correction is applied in the receiver's navigation algorithm as part of the relativistic adjustment to the pseudo range measurements. Unlike the net frequency shift (which is handled by a fixed pre-launch offset), the Sagnac correction varies with time and must be computed in real time for each satellite in view. The correction values are given below:

Geometry	Approximate Δt_{Sag}
Satellite near equator, receiver on equator, eastward line-of-sight	+90 ns
Satellite near equator, receiver on equator, westward line-of-sight	-90 ns
Satellite overhead, receiver at mid-latitude	$\approx 50 \text{ ns}$
Satellite near horizon, receiver at pole	$\approx 0 \text{ ns}$ (since $x_R y_S - y_R x_S$ is small)

The sign of the correction depends on the relative east-west orientation: if the satellite is to the east of the receiver in the ECEF frame, the correction is positive (signal takes longer).

10.1. Effect on Positioning: Since light travels about 0.3 m per nanosecond, a timing error of 100 ns translates to a range error of 30 meters. If the Sagnac effect were ignored, GPS would produce systematic errors of the order of 10–30 meters, which is unacceptable for modern navigation (especially for aircraft landing, autonomous vehicles, or surveying). The Sagnac correction is therefore mandatory in every GPS receiver's positioning algorithm.

11. Relativity and GPS synchronization

GPS requires a common reference time for all satellites. This synchronization cannot be treated simply as if all clocks were stationary in an inertial laboratory. The Earth is rotating, the satellites are moving, and the gravitational potential varies with position. Consequently, GPS uses a carefully defined coordinate-time framework in which satellite clock readings are related to a common system time. The success of GPS demonstrates that relativistic concepts of time, synchronization and reference frames are necessary for modern global navigation. Ashby describes GPS as a particularly important practical application of special and general relativity.

12. Broader Applications

Relativistic corrections are required not only for GPS but also for other satellite navigation systems, also when comparing atomic clocks at different locations. Precise measurements of Earth's shape and gravitational field require accurate timing. Relativistic effects become increasingly important for navigation over large distances and high-precision missions. Satellite clocks provide opportunities for testing theories of gravity and searching for deviations from general relativity.

13. Conclusions

In summary, this work has reviewed the theoretical foundations, experimental validations, and practical implementations of relativistic time dilation—both kinematic from Special Relativity and gravitational from General Relativity with particular emphasis on their indispensable role in the Global Positioning System (GPS). The core findings can be summarized as follows: Time is not absolute. Einstein's Special Relativity (1905) and General Relativity (1915) established that the rate at which time passes depends on relative motion and gravitational potential. This counter intuitive conclusion—now confirmed by countless experiments is not merely a theoretical curiosity but a practical necessity for modern technology. Kinematic time dilation (moving clocks run slower) and gravitational time dilation (clocks in weaker gravity run faster) both affect GPS satellites. For a typical GPS satellite at $\approx 20,200$ km altitude, the gravitational blue shift ($+45.7 \mu\text{s/day}$) dominates over the kinematic redshift ($-7.2 \mu\text{s/day}$), yielding a net gain of approximately $+38.6 \mu\text{s/day}$. Without correction, this would cause positioning errors of about 11 km per day, making GPS unusable within hours. Relativistic corrections are indispensable for accurate GPS positioning. In addition to the two principal time-dilation effects, precise GPS operation requires corrections associated with orbital eccentricity, Earth's rotation and the Sagnac effect, as well as other smaller relativistic contributions.

REFERENCES:

1. D. Halliday, R. Resnick, and J. Walker, 2013, Fundamentals of Physics, 10th edition, John Wiley & Sons, 5 August: 1456.
2. Einstein, 1915, "Die Feldgleichungen der Gravitation," Sitzungsber. Preuss. Akad. Wiss. Berlin, pp. 844–847.
3. Einstein, 1905, "On the Electrodynamics of Moving Bodies," Ann. Phys., vol. 17: pp. 891–921.
4. Encyclopedia Britannica, 2024; "Time dilation," <https://www.britannica.com/science/time-dilation>.
5. NIST Time and Frequency Division. (2019). Putting Einstein to the Test with the World's Most Accurate Clocks. Taking Measure Blog. Available: <https://www.nist.gov/blogs/taking-measure/putting-einstein-test-worlds-most-accurate-clocks>
6. Einstein, A. (1916). "Die Grundlage der allgemeinen Relativitätstheorie" (The Foundation of the General Theory of Relativity). Annalen der Physik, 49, 769–822.
7. Will, C. M. (2014). "The Confrontation between General Relativity and Experiment." Living Reviews in Relativity, 17(1), 4.
8. R. V. Pound and G. A. Rebka Jr., 1960 "Apparent Weight of Photons," Phys. Rev. Lett., vol. 4, no. 7, pp. 337–341, Apr., doi: 10.1103/PhysRevLett.4.337.
9. N. Ashby, "Relativity in the Global Positioning System," Living Rev. Relativity., vol. 6, no. 1, 2003, doi: 10.12942/lrr-2003-1.
10. Bothwell, T., Kennedy, C. J., Aeppli, A., Kedar, D., Robinson, J. M., Oelker, E., Staradonova, A., & Ye, J. (2022). Resolving the gravitational redshift across a millimetre-scale atomic sample. Nature, 602(7897), 420–424.
11. J. C. Hafele and R. E. Keating, 1972, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains," Science, vol. 177, no. 4044, pp. 166–168, Jul. doi: 10.1126/science.177.4044.166.

12. Cacciapuoti, L., et al. (2024). Atomic Clock Ensemble in Space. arXiv preprint arXiv:2411.02912